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METHODS OF LOGICAL, RECURSIVE AND OPERATOR ANALYSIS AND SYNTHESIS OF AUTOMATA

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An automat may be regarded as an operating simulator of a system interacting with the environment and, consequently, as a simulator of a discrete control process. The study and solution of the problem of simulating a real system is a complex process, which involves: the selection of essential characteristics of the system and its interaction with the environment; the approximate description of the system by means of the totality of characteristic features, either algorithm, or structure; the analysis of the obtained information and its transformation into an optimum system of logical functions; the synthesis of a real automat using a certain system of physical components, executing logical functions.

Another alternate approach of the problem is the simulation of a real system (or control process) in the form of a process in an automat with a preset structure, endowed with many degrees of freedom.

In this case, it is desirable that the analysis of the system data be reduced to a certain optimum algorithm and dismember it into independent blocks executed in the automat by means of logical circuits or a program.

The development of digital computer logics and the employment of the machine for handling the information belong to problems of this kind.

The solution of the problem of analysis and synthesis of automata may be performed on the basis of different formal descriptions.

The present paper deals with three alternate approaches to this problem: the calculation of logical functions of time, the analytical medium for the presentation of recursive functions and the operator representation of the process in the automat.

Para.1. AUTOMATA AND LOGICAL FUNCTIONS OF TIME

As an analytical medium for the presentation and analysis of logical features we use the calculus of logical time functions (1), pressupposing that the processes of data conversion occur discretly in time while their description is related to the current moment, the origin.

From here takes its source the method of the process prehistory description, expressed in terms of time delays. To the zero operations on two-valued variables is added the operation of time shift ($D^{\kappa}x$) and relation of logical inequality ($x \leq y$, $x < y$).

In relation to any formula, built-up of two-valued variables by means of introduced operations, the time shift responds to the distribution law.

Over words of equal length, composed of two-valued variables are introduced interdigit operations of negation ($\bar{X}$) crossover ($X \cap Y$) conjunction ($X \cup Y$) and time shift ($D^{\kappa}X$). Besides, the operation is used for cycle shift of the word over a preset number of digits to the left, or to the right ($R_o^{\kappa}X$) and of the relation of logical inequality ($X \leq Y$, $X < Y$). In relation to any formula, built up of words of equal length by means of introduced operations, the time shift and cycle shift respond to the law of distribution.

For characterizing the common features of the word components, are used the operations of conjunction and disconjunction convolution of the word ($L^{\wedge}X$, $L^{\vee}X$).

The time function f (X) from n two-valued arguments $x_1, \ldots, x_n$ composing the word X, may be generated into the normal disconjunction form and expressed in operator form by means of the generating word B, with a length of 2^n, according to the definition

$$f(X) \sim R^{\vee}(B,X) \sim \bigvee_{j=0}^{2^n-1} [L_j B \wedge L^{\wedge}(A_j \cap X)]$$

Each formula ("the original") F ($x, y, \cdots$) on two-valued variables may be compared to the formula ("image") F^* ($X, Y, \ldots$) on words of equal length; The structure of this formula is the same as that of the original, but instead of the time shift operations it contains word cycle shift operations. It may be demonstrated that for any function F over time polynomials from X, not containing time shifts, the distribution law is in force.

$$F[R^{\vee}(B_1,X),\ldots,R^{\vee}(B_\kappa,X)] \sim R^{\vee}[F^*(B_1,\ldots,B_\kappa),X]$$

Thus, to a great extent the transformation of functions from two-valued variables, may be reduced to the transformations of the generating words of their normal polynomials.

At the description and analysis of the features of discrete processes, the logical functions are often formed in an implicit form of logical time equations, of the kind

$$F(Z) \sim R^{v}(B, Z) \sim C_0$$

where the components of the word Z are two-valued variables $\alpha_1, \ldots, \alpha_K$ and their time shifts $D^{0}\alpha_{\pi}, \ldots, D^{m-1}\alpha_{\pi}$, a C_0 is the logical constant (zero).

Such an equation in implicit form defines in the general case the family of logical functions in explicit form

$$\alpha_{\pi} \sim f_{\pi}(Z_{\pi}) \sim R^{v}(B_{\pi}, Z) \qquad \pi = 1, \ldots, K$$

where Z_{π} does not contain $D^{0}\alpha_{\pi}$. The general method of solution / 2 / of time logical equations (the reduction method) leads to a set of inequalities for the generating words of the B_{π} polynomials of the functions to be determined

$$K_{\pi} \leq B_{\pi} \leq M_{\pi}, \qquad \pi = 1, \ldots, K$$

where K_{π} and M_{π} are functions of B and of the adopted order of variable reduction. Depending upon the degree of redundancy of the initial equation and of the order of variable reduction, the limits of admissible values for each B_{π} may be more or less narrow. Thus, for each particular solution is defined the character of the variables and the degree of their dependence from the other variables. In all cases, when solving the equation, we obtain a set of functions corresponding to correctly organized logical nets.

Let us consider the methods of possible mappings between sets of time logical functions and sets of recursive functions.

Take, for instance, a set of time logical functions $z_1, \ldots, z_n$ from two-valued variables $x_1, \ldots, x_m$. Let, for each two-valued variable, the " u " numerical value be $N(u) = 0$ or 1 ; besides, we determine for elementary operations

$$N(\bar{u}) = 1 - N(u), N(u \wedge v) = N(u) \cdot N(v), N(0^{K}u) = 0^{K} N(u)$$

Attributing to the words

$$X = \Gamma_{i=1}^{m} x_i \quad and \quad Z = \Gamma_{i=1}^{n} z_i$$

concrete weighting position functions

$$N(X)=\sum_{i=1}^{m} f_x(i)\, N(x_i), \quad N(Z)=\sum_{i=1}^{n} f_z(i)\, N(z_i)$$

we obtain the mapping of the set of time logical functions in the form of a special set of recursive functions defining the number $N(Z)$ as the function of the number $N(X)$ and some precedent values of the numbers $N(X)$ and $N(Z)$.

In this manner may be formed many different numerical interpretations of the given set of logical functions.

The mapping of a given set of recursive functions on a certain special set of logical functions also allows to obtain many solutions depending on the adopted arithmetic system, the limits of modification of numerical parameters and the auxiliary time conditions set for the formation of new parameter and function values.

No general method of transfer from the numerical recursive function to logical functions, has apparently yet been developed.

However, in many cases it is possible to specialize the record of a preset algorithm in recursive form so that only a standard set of recursive functions-components is used, to which a set of standard time logical functions corresponds. In these cases, the logical function and the circuit for the execution of the given algorithm may be easily formed.

As known, this method of approach to the problem is used in designing of machines for information processing.

Now let us consider the conception of the automat and describe it by means of logical functions of time.

Take a square q-dimensioned matrix $\| x_{ij} \|$ with components which are the time polynomials of a predetermined set of external arguments $U_1, \ldots, U_K$. The matrix is called true, when each filled column possesses the feature that any column element is a complement to the sum of the rest of the elements with respect to 1.

We employ the true matrix $\| x_{ij} \|$ for defining the finite automat with a "q" states $y_1, \ldots, y_q$, assuming that the current value of the matrix element x_{ij}, determines the transition from the state y_j to the state y_i

with a time delay equal to one unit, according to the equation:

$$y_i \sim \bigvee_{j=1}^{q} [D' y_j \wedge x_{ij}], \quad i=1,\dots,q$$

If at a certain moment only one of the variables $y_1, \dots y_k$ was equal to 1, this feature remains unchanged at any subsequent moment. This means that in the mentioned conditions the set of states is full, and the states themselves are incompatible in pairs; such an automat possesses the properties of single-value and continuity of transfers.

The output functions of the automat $z_1, \dots, z_p$ should be, naturally, defined as polynomials of the current values of the state; in view of the incompatibility of these states in pairs, these functions are reduced to some disjunctions of the variables $y_1, \dots, y_q$.

The aim of the described conception of the automat is to isolate from the automat structure, the primitive (containing no feed backs) functions of external arguments.

If the automat is determined by a transition matrix, its logical structure is directly determined by the afore-indicated set of time logical equations and an adequate correctly organized logical net. The solution of the reverse problem of designing a matrix of automat transitions for a predetermined set of time logical functions, involves the substitution of variables, the solution in relation to new variables of generalized equation in implicit form and the subsequent determination of the matrix elements. Various alternative solutions of the generalized equation lead to different automata which are equivalent in that the same set of output functions corresponds to them.

From each given automat may be formed many equivalent automata either by splitting some states, or by duplicating the group of coupled states (cells). The inverse process consists in the compression of the automat structure by bringing together separate states or cells, on condition that the output function is conserved. This results in a certain minimum structure of the automat / 3 /.

It may be anticipated that the further development of the methods of equivalent transformation of automata structures will be one of the efficient methods of approach to the analysis and synthesis of classes of automata endowed with certain particular features.

Para.2. AUTOMATA AND RECURSIVE FUNCTIONS

Let us consider the description of the automat as a device for the realization of a set of recursive functions. Such a description is especially convenient for automata realizing computing algorithms. We assume that the automat comprises a set of digit registers (cells): $\alpha^1, \alpha^2, \dots, \alpha^m$ each of which, at a given step of operation, contains a certain number designated further as $[\alpha^i]$ or $[\alpha^i]_n$ and where n - is the number of the step. The state of the automat is determined by the set of numbers $[\alpha^1], [\alpha^2], \dots, [\alpha^m]$

We shall assume that part of the registers $\alpha^{m+1}, \alpha^{m+2}, \dots \alpha^N$ are input registers, i.e., are such the content of which is determined by the information coming from outside. The state of all other registers at the $n+1$- step is determined by the contents of all the registers at the preceding n -step,

$$[\alpha^i]_{n+1} = \mathcal{F}_i([\alpha^1]_n, [\alpha^2]_n, \dots, [\alpha^m]_n, [\alpha^{m+1}]_n, \dots, [\alpha^N]_n) \quad (1)$$

$$i = 1, 2, \dots, m$$

The set of functions $\mathcal{F}_i$ characterizes in full the automat structure. The task of the synthesis is to design, taking as a basis a certain class of algorithms - an automat with such a set of determining functions $\mathcal{F}_i$ which should permit to realize these algorithms / 4 /.

Let us analyze what does represent the set of determining functions $\mathcal{F}_i$ for a three-address computer operating on the position code principle.

Let α^1 - be the register storing the command to be executed,
α^2 - the register storing the address of the command which will be executed in the next step, and $\alpha^3, \alpha^4, \dots, \alpha^m$ - the operational memory cells.

We do not deal with the input registers considering that at the initial moment all the input information enters into the operational memory.

During each step, there is to be modified only the contents of the cell corresponding to the third address of the command to be executed. For the case when the contents of a certain cell α, is a command, we shall adopt the following designations.

The part of the α cell contents, which is the code (symbol) of the command, we shall designate by $[\alpha]^{CK}$, and the parts $[\alpha]$ which are the first, second and third addresses we shall designate by $[\alpha]^{I}$, $[\alpha]^{II}$ and $[\alpha]^{III}$ respectively. Then we introduce the function $S(a,b)$ which is equal to zero at a $a \neq b$, and equal to 1 at a $a = b$, and the function $\bar{S}(a,b) = 1 - S(a,b)$. If the executed command is not a command of control transfer, the set of relations (I) may be written as follows

$$[\alpha_1]_{n+1} = [[\alpha_2]_n]_n ,$$

$$[\alpha_2]_{n+1} = [\alpha_2]_n + 1, \qquad (2б)$$

$$[\alpha]_{n+1} = [\alpha]_n \bar{S}(\alpha, [\alpha_1]_n^{III}) + S(\alpha, [\alpha_1]_n^{III}) \sum_{\gamma,\beta,\Phi} \Phi([\gamma]_n, [\beta]_n) S(\gamma, [\alpha_1]_n^{I}) S(\beta, [\alpha_2]_n^{II}) \cdot S(N\Phi, [\alpha_1]_n^{CK}) \qquad (2c)$$

We assume that the three-address command is executed in the following manner: $\Phi([\alpha]_n, [\beta]_n) \rightarrow \gamma$ where $\Phi(x, y)$ designates an arbitrary operation executed with the numbers x and y during one step by the computer arithmetic device, and $N\Phi$ - the code (symbol) of the corresponding command. In the sum (2B) only one addend differs from zero, for which $\alpha = [\alpha_1]_n^{I}$ and $\beta = [\alpha_1]_n^{II}$, i.e., the operation $\Phi(x, y)$ is executed with data stored at the addresses determined by the command in the register α_0. The presence of summation in all the pairs of cells, (α, β) according to the formula (2B), involves the necessity of full scanning in the memory for selecting the required cells. Let us consider the case when an execution of the control transfer command is possible.

Let us designate by $\Phi_0 \alpha\beta\gamma$ the command of conditional transfer, which has the following meaning: if $[\alpha] > [\beta]$ then the control is transferred to the following by order command, while if $[\alpha] \leq [\beta]$, the control is transferred to the command in the cell γ. Similarly let us designate the command of unconditional transfer by $\Phi_1 \alpha\beta\gamma$ which in all cases transfers the control to the command $[\gamma]$.

At the presence of these commands, the equality (2a) is kept, while the equalities (2б) and (2c) are complicated in the following way. (2б) is replaced by the equality

$$[\alpha_2]_{n+1} = \{[\alpha_2]_n + 1\} S([\alpha_1]_n^{CK}, \Phi_0)\{1 - \mathrm{sign}([\alpha_1]_n^{II} - [\alpha_1]_n^{I})\} + \{[\alpha_2]_n + 1\} \bar{S}([\alpha_1]_n^{CK}, \Phi_0) S([\alpha_1]_n^{CK}, \Phi_1) + \{S([\alpha_1]_n^{CK}, \Phi_0)\,\mathrm{sign}([\alpha_1]_n^{II} - [\alpha_1]_n^{I}) + S([\alpha_1]_n^{CK}, \Phi_1)\}[\alpha_1]_n^{III} \qquad (2б')$$

In order to suitably modify the formula (2c) we designate, for abbreviation, the right-hand part of the equation (2c) as G_α.

Then in the considered case we shall have

$$[\alpha]_{n+1} = G_\alpha \{1 - S([\alpha_1]_n^{ck}, \varphi_0) - S([\alpha_1]_n^{ck}, \varphi_1)\} + + [\alpha]_n \{S([\alpha_1]_n^{ck}, \varphi_0) + S([\alpha_1]_n^{ck}, \varphi_1)\} \qquad (2c)$$

This means that if the command $[\alpha_1]$ is a transfer command, then the contents of all memory cells remain as before, i.e.,

$$[\alpha]_{n+1} = [\alpha]_n$$

If the machine contains group operations and other control operations, the set of equalities (2) becomes even more complicated. For the computing algorithm preset in the B operator form [5] it is possible to form a set of functions (I) which realizes this algorithm. This representation is convenient in that the set (I) is directly connected with the structure of the computer (automat) and at the same time is B of "big-block structure". This means that the recursive recording of the automat dynamics does not take into consideration, for instance, the peculiarities of the structure of single digit adders or the methods of multiplication acceleration, while the description of the automat in terms of time logical functions would essentially include these features.

In terms of functions $\mathcal{F}_i$ it is possible to characterize the complexity of the algorithm as well as of the automat realizing this algorithm. (Recently A.P. Ershov (6) has indicated the relation of computing algorithms with the recursive functions).

The complexity of the automat is characterized by the complexity of the set type (2), i.e., by the number of registers with simultaneously changing contents, the number of registers determining the modification of the contents in each register etc. The complexity of the algorithm is characterized by the complexity of the automat realizing the given algorithm during a certain number of steps.

If the system set of recursive relations (1) is expressed in the form of equalities of the type (2), it is possible to build up a set of time logical functions realizing this set of relations. It is only to be remembered that at the transfer to time logical functions it is sometimes expedient to reduce the time scale.

For computing machines a very important point is the choice of an efficient structure of the control device, ensuring a maximum capacity at the given high speed operation of the arithmetic circuits. With this aim in view it is necessary to design such circuits that may conveniently realise complicated combinations of operations for the conversion resetting and formation of addresses. Let us consider the class of recursive functions (I) in which the functions $\mathcal{F}_i$ are combinations of the following basic functions:

I. Function $S(x,y)$ determined by the equality

$$S(x,y) = \begin{cases} 1 \text{ at } x = y \\ 0 \text{ at } x \neq y \end{cases}$$

2. The function $\text{sign}_1 x$ and $\text{sign}_2 x$ determined by the equalities

$$\text{Sign}_1 x = \begin{cases} 1 \text{ at } x \geq 0 \\ 0 \text{ at } x < 0 \end{cases} \qquad \text{Sign}_2 x = \begin{cases} 1 \text{ at } x > 0 \\ 0 \text{ at } x \leq 0 \end{cases}$$

3. The operation of adding a unit: $x+1$

4. The operation of addition $x+y$ modulo 2^{ℓ}

5. The operation of subtraction $x-y$ modulo 2^{ℓ}

6. The function $\pi(x,y,z)$ executing the counting by modulo y with resetting to initial x-value

$$\pi(x,y,z+1) = \begin{cases} \pi(x,y,z)+1 \text{ if } \pi(x,y,z) \neq y \\ x \qquad\quad \text{ if } \pi(x,y,z) = y \end{cases}$$

The function $\pi(x,y,z)$ may be expressed by the preceding functions, but it plays an important role in the synthesis of the program, and therefore we use for this function a special designation.

7. The operation of formation over any group of arguments $\varphi_p(x,y,z,\dots,\omega)$ at which the words representing the values $x,y,z,\dots,\omega$ are written from left to right, thus forming the resulting word.

We shall combine the functions enumerated above by substituting the arguments by functions. At the same time we shall admit the so-called "time transformation" according to the following rule. We assume that the set of recursive functions $f_n^1, f_n^2, \ldots, f_n^s$ has been determined and another set of recursive functions has been formed from the variable $\mathcal{N}$:

$$\varphi_{\mathcal{N}}^1, \varphi_{\mathcal{N}}^2, \ldots, \varphi_{\mathcal{N}}^t$$

Then the set of the functions,

$$f_{\varphi_{\mathcal{N}}^1}^1, f_{\varphi_{\mathcal{N}}^1}^2, \ldots, f_{\varphi_{\mathcal{N}}^1}$$

we shall designate as the set obtained by "time transformation". This kind of procedure is very useful for building additional cycles of lower degree into the program.

Recursive functions originating from this class allow to record conveniently a number of computing algorithms. As an example may be cited the recording of an algorithm for the solution of a set of linear equations

$$x_i = \sum_{k=1} a_{ik} x_k + b_i \qquad (3)$$

by the Seidel iteration method.

$$x_i^{(s+1)} = \sum_{k=1}^{i-1} a_{ik} x_k^{(s+1)} + \sum_{k=i}^{\mathcal{N}} a_{ik} x_k^{(s)} + b_i \qquad (4)$$

breaking off at the condition

$$\max \left| x_i^{(s+1)} - x_i^{(s)} \right| < \varepsilon \qquad (5)$$

$$\text{or } S = S_0$$

Let the addresses of the coefficients a_{ik} be $(i-1)\mathcal{N} + k - 1 + \xi_0$, the addresses of the values in S approximation $(x_i^{s)}) = \xi_1 + i - 1$, and the addresses of the values in the $S+1$ approximation $(x_i^{(s+1)}) = \xi_2 + i - 1$. The addresses of the free members are chosen in form of

$$(b_i) = \xi_3 + i - 1$$

Now, the calculations according to the formula (4) are ensured by the commands

$$\left.\begin{array}{llll} (1) & y\varphi(\mathcal{N}-1) + k - 1 + \xi_0 & \xi_8 + k - 1 & \beta_0, \\ (2) & C\alpha\beta_0 & \beta_1 & \xi_2 + i - 1, \end{array}\right\} (6)$$

where $\delta = 2$, if $i < K$ and $\delta = 1$ if $i \geq K$

The verification of the conditions (5) is made by means of the commands

$$\begin{array}{lllll} (1) & B\tau_2 & \xi_1 + i - 1 & \xi_2 + i - 1 & \beta_2 \\ (2) & B\tau_1 & \beta_2 & (\varepsilon) & \xi_1 + i - 1 \end{array} \qquad (7)$$

and by checking whether all the values $[\xi_1 + i - 1]$ are negative. Here $B\tau_1$ means ordinary subtraction, and $B\tau_2$ the formation of the difference absolute value.

First, let us obtain the recursive functions determining the addresses of the commands in conventional time. The calculations according to the commands (7) shall be made as the values in $S+1$ approximation are computed according to the commands (6).

We introduce the cells $\gamma_1, \gamma_2, \gamma_3, \gamma_0$ for three variable addresses in the commands (6) and the cells $\gamma_4, \gamma_5, \gamma_6$, storing the number of the factor K in the sum (4), the number of the line i , and the number of the executed S iteration respectively. The addresses in (7) are stored in the cells γ_0 and γ_* . The contents of these cells are determined as a set of recursive functions of the following form:

$$\left.\begin{array}{l}
[\gamma_4]_n = \pi(1, N, n), \\
[\gamma_5]_{n+1} = \{[\gamma_5]_n + S([\gamma_4]_n, N)\}(1 - S([\gamma_5]_n, N)) + S([\gamma_5]_n, N), \\
[\gamma_5]_0 = 1 \\
[\gamma_6]_{n+1} = [\gamma_6]_n + S([\gamma_5]_n, N), \\
[\gamma_6]_0 = 0. \\
[\gamma_1]_n = \pi(\xi_0, \xi_0 + N^2, n), \\
[\gamma_2]_n = \pi(\xi_1, \xi_1 + N, n), \\
[\gamma_3]_n = \pi(\xi_2, \xi_2 + N, n), \\
[\gamma_0]_0 = \xi_2 \\
[\gamma_0]_{n+1} = ([\gamma_0]_n + S(N, [\gamma_4]_n))(1 - S([\gamma_5]_n, N)) + S([\gamma_5]_n, N)\xi_2, \\
[\gamma_*]_0 = \xi_1, \\
[\gamma_*]_{n+1} = ([\gamma_*]_n + S(N, [\gamma_4]_n))(1 - S([\gamma_5]_n, N)) + S([\gamma_5]_n, N)\xi_1
\end{array}\right\} (8)$$

We introduce the function $[\varphi]_n$ which takes the value I in case the iterations are completed, and zero, in the opposite case. From the conditions in which the iteration process breaks up may be seen that:

$$[\varphi]_n=\{1-\prod_{i=1}^{\mathcal{N}}(1-Sgn[\xi_1+i-1]_n)\}Sgn(S_o-[\gamma_6]_n)+1-Sgn(S_o-[\gamma_6]_n) \qquad (9)$$

Now, for the command executed at the moment ν we can record

$$\begin{aligned}[\alpha_o]_\nu=\{&S([\alpha_1]_\nu,1)\{\varphi_p(y[\gamma_1]_n,[\gamma_2]_n,\beta_o)Sgn_1([\gamma_5]_n-[\gamma_4]_n)+\\ &+\varphi_p(y,[\gamma_1]_n,[\gamma_3]_n,\beta_o)Sgn_1([\gamma_4]_n-[\gamma_5]_n)\}+\\ &+S([\alpha_1]_\nu,2)\varphi_p(C_A,\beta_o,\beta_1,[\gamma_o]_n)+\\ &+S([\alpha_1]_\nu,3)\varphi_p(Br_2,[\gamma_*]_n,[\gamma_o]_n,\beta_2)+\\ &+S([\alpha_1]_\nu,4)\varphi_p(Br_1,\beta_2,(\varepsilon),[\gamma_*]_n)\}(1-[\varphi]_n)+[\varphi_n]Cm.\end{aligned} \qquad (10)$$

It remains to determine the function $[\alpha_1]_\nu$ and effect the "time transformation". The function $[\alpha_1]_\nu$ is determined by the conditions

$$[\alpha_1]_o=1$$

$$\begin{aligned}[\alpha_1]_{\nu+1}=&\{([\alpha_1]_\nu+1)S([\alpha_1]_\nu,1)+S([\alpha_1]_\nu,2)\}(1-S([\gamma_1]_n,\mathcal{N}))+\\ &+S([\gamma_1]_n,\mathcal{N}\{S([\alpha_1]_\nu,4)+([\alpha_1]_\nu+1)(1-S([\alpha_1]_\nu,4))\}\end{aligned} \qquad (11)$$

Finally, the "time transformation" is determined by the function determined by the conditions $n=\Psi_\nu,\ \Psi_o=0$

$$\Psi_{\nu+1}=\Psi_\nu+(1-S([\gamma_1]_n,\mathcal{N}))S([\alpha_1]_n,2)+S([\gamma_1]_n,\mathcal{N})S([\alpha_1]_\nu,4) \qquad (12)$$

The set of relations (8), (9), (10), (11), (12) fully presets the program of calculations according to the Seidel method. These considerations form a sound basis for the choice of the control device structure. On similar considerations may be based the choice of the memory device structure and, in particular, the choice of the system of a multistage memory and for the use of the memory with successive read out.

Para. 3. AUTOMATA AND PROGRAM OPERATORS

Let us consider the finite automat A, the state of which is characterized by the states "n" of its components e_i $(i=1,2,\ldots,n)$. Let us assume that $\{n\}$ is the set of all integers from I to n. Let us consider the states at a certain moment of time, as an aggregate of values of a certain function f on a set of $\{n\}$; Thus it can be said that A at the given moment is in the state f. Since we are speaking of a discrete A, we may assume a certain "τ" which is the minimum necessary for the transition of A from one distinguishable state f_1 into the other state f_2. Let us designate τ - as the A step. We shall call by the term "command" the different operations that is capable of executing during τ. Let A_0 be at a certain moment in the state $f_{(0)}$. As a result of the step performed, A has passed from the state $f_{(0)}$ into the state $f_{(1)}$. The operator K_1 is such that $f_{(1)}=K_1 f_{(0)}$ is the command oper- /7/.

Suppose, that for each e_i there is some zero stat- the state A will be zero (0) if all the e_i are in zero state. The zero-operator is such that $Of=O$. The unit operator E determined from $Ef=f$ is the idle step operator.

If in A pass consecutively steps with operators $K_1, K_2, \ldots$, the A will consecutively be in the states

$$f_{(1)}=K_1 f_{(0)},\ f_{(2)}=K_2 K_1 f_{(0)},\ \ldots,\ f_{(j)}=K_j K_{j-1}\cdots K_1 f_{(0)}$$

Usually the operation of the automat is determined by a certain finite number of commands (the program), which are reproduced consecutively (the cycle). If the operators of these programs are K_j $(j=0,1,\ldots,m)$, then P - the program operator - is equal to

$$P=K_m K_{m-1}\cdots K_2 K_1 \qquad (I).$$

Let F_0 be the starting state. As a result of the operation of A, shall be obtained the states $F_j=P^j F_0$ $(j=1,2,\ldots,S)$. In such a manner operates a homogeneous automat, i.e. without any external modification of its states. There might be homogeneous automata, in which in the S cycle, after the operator K_{t_s} the state φ_S is exteriorly set, by means of the input operator G_S. Then, in the A are obtained the following states:

$$F_S=P_{t_S,m}\left[G_S(P_{1,t_S}F_{S-1},\varphi_S)\right], \qquad (S=1,2,\ldots)$$

where $P_{t_S,m}=K_m\cdots K_{t_S+1}$, $P_{1,t_S}=K_{t_S}\cdots K_2 K_1$.

Two types of input operators are to be considered that is

1. When the introduced states are superposed on the states of A at the moment of input. Then

$$F_S = P_{t_S, m}[P_{1,t_S} F_{S-1} + \varphi_S]$$

2. When the introduced states replace the states of the A components at the moment of input. Then

$$F_S = P_{t_S, m}[P_{1,t_S} F_{S-1} + E_{\{\nu_S\}}(\varphi_S - P_{1,t_S} F_{S-1})]$$

where $\{\nu_S\}$ is the set by which f_S differs from zero (i.e. the states of the elements e_i with numbers from $\{n\}-\{\nu_S\}$ remain unchanged), and $E_{\{\nu_S\}}$ is the projection of the operator E over the set $\{\nu_S\}$ ($E_{\{\nu_S\}} f = f$ over a set $\{\nu_S\}$ and $E_{\{\nu_S\}} f = 0$ in the points $\{n\}-\{\nu_S\}$).

The automat synthesis problem may beraised as follows: There are given - the operator U, the function $f_{(0)}$ and the commands aggregate $\{K\} = K_1, K_2, \dots, K_j, \dots, K_m$ On the basis of the operators from $\{K\}$ we must build-up an automat passing through the states $F_S = U^S f_{(0)}$ Let $\varphi_{(0)}$ be the starting state of A and P - its program operator and let A solve the problem. Then, presumably the following relations are to be executed

$$P^S \varphi_{(0)} = U^S f_{(0)} \qquad (s = 1, 2, \dots)$$

Generally speaking, if U and $f_{(0)}$ are not in a certain way specialized, only $\varphi_{(0)} = f_{(0)}$ and $P = U$ are possible. This means, that at such a statement of the problem, the operator U must belong to the class of operators which are factorized into the product of command operators from $\{K\}$. However, the problem of synthesis acquires a greater interest in case when U either cannot at all be factorized into the product of operators from $\{K\}$ or it is factorized into the product obtained from a great number of such operators, which leads to the synthesis of the automat involving a great time of operation.

Therefore, it is expedient to bring to certain modification in the statement of the synthesis problem, and demand that the states of A coincide with the required states not for all components, but only for certain preset components. In other terms, the functions characterizing these states must coincide over a certain set $\{\nu\} \subset \{n\}$.

If we designate with $\mathcal{E}_{\{\nu\}}$ an operator which ensures that at any function "f" there will be $\mathcal{E}_{\{\nu\}} f = f$ over a set $\{\nu\}$ and $\mathcal{E}_{\{\nu\}} f = \xi$ over a set $\{n\} - \{\nu\}$ (ξ -is an arbitrary function), then for the synthesis P it shall suffice to realize the relation $P\mathcal{E}_{\{\nu\}} = \mathcal{E}_{\{\nu\}} U$ (2). Then, $\varphi_{(o)} = \mathcal{E}_{\{\nu\}} f_{(o)}$.

From this equation are to be determined P and $\mathcal{E}_{\{\nu\}}$. Let us consider now the statement of the synthesis problem for the case of a non-homogeneous automat. Additionally to the problem of the homogeneous automat synthesis for obtaining the required states, it is admitted that into each operation cycle of the automat is introduced from the outside of the state ψ.

For better certainty, let us assume that ψ is introduced at the end of the cycle. Let $\varphi_{(o)} = \mathcal{E}_{\{\nu\}} f_{(o)}$ and $\psi = R f_{(o)}$ For the input, according to the type 2, with the operator $E_{\{r\}}$ it will suffice to realize the relations:

$$E_{\{n\}-\{r\}} P\mathcal{E}_{\{\nu\}} - \mathcal{E}_{\{\nu\}} U = -E_{\{r\}} R \qquad (3)$$

at the condition superposed on the operator R - the condition of the invariability of the state ψ introduced from outside

$$E_{\{r\}}(RU - R) = 0 \qquad (4)$$

for the input according to type I these relations take the following form:

$$P\mathcal{E}_{\{\nu\}} - \mathcal{E}_{\{\nu\}} U = -R \qquad (5)$$

$$RU - R = 0 \qquad (6)$$

These relations have determined a certain class of operators U for which may be synthesized the automat on the basis of operators $\{K_j\}$. Let us consider some possibilities of extending this class. Assume that T and M, operators of the type $\mathcal{E}_{\{\nu\}}$ -are commutating operators, with U and P respectively. In this case, for the synthesis of a homogeneous automat, with the adopted statement of the problem, it is sufficient to meet the relation

$$MP\mathcal{E}_{\{\nu\}} = \mathcal{E}_{\{\nu\}} TU$$

For the synthesis of a nonhomogeneous automat the respective relations shall be written as follows

$$M(P\mathcal{E}_{\{\nu\}} + R) = \mathcal{E}_{\{\nu\}} TU \qquad (7)$$

$$MR - RTU = 0 \qquad (8)$$

The operators M and T, as extending the possibilities of satisfying the conditions (2) - (6) for the synthesis of the automat may be called "automation multipliers" M - left-hand and T - right-hand multipliers. The multipliers T and M may be introduced in corresponding conditions both separately and together. The introduction of the multiplier T sets the following limitation on U :

U must belong to a class (let us name it the IId class) of operators which admits commutating operators of the type $\mathcal{E}_{\{\nu\}}$. Of considerable interest from the point of view of automata synthesis is the introduction of left-hand multipliers. In this case, it is not obligatory for the operator U which is preset for realization to belong to the IId class, but this feature must necessarily possess the operator of the automat P. It seems expedient to differentiate the automata with operators belonging to the IId class. We may call such automats as high-quality automata - for they present greater facilities for meeting the synthesis conditions. The quality of such automata depends upon the arbitrariness in M. The greater the arbitrariness allowed in the determination of M for the given P the more are the possibilities to direct this arbitrariness for the satisfaction of the corresponding synthesis conditions.

We may discuss the question of satisfying the synthesis conditions in the sense of expanding the set $\{n\}$, over which is determined the preset operator U up to $\{n+m\}$. Here, is essential the conception of effective expansion of the operator U up to $\bar{U}$, i.e. such an expansion that $\mathcal{E}_{\{n\}}\bar{U}$ coincides with U. The problem of the automat synthesis with $\{n+m\}$ components for the realization of $\bar{U}$ is equivalent to the problem of the automat synthesis for the operator U. In this case into the automat are introduced auxiliary components, the operation of which aims at satisfying the synthesis conditions.

The purpose of the above considerations was to satisfy by means of various transformations the conditions of the synthesis, by automata, having program operators P which may be thoroughly studied. For the synthesis conditions it is not sufficient to know P, but it is necessary also to know the starting state $\varphi_{(o)}$. The solution of the equations (2), (3), (5) in relation to $\mathcal{E}_{\{\nu\}}$ allows to determine $\varphi_{(o)} = \mathcal{E}_{\{\nu\}} f_{(o)}$. However, it is a complicated procedure. A more simple method may be proposed. Let us determine the functions $f_{(i)} = U^i f_{(o)}$. Assume that

$$\{n\} = \{\nu\} + \{\nu_1\} + \cdots + \{\nu_s\}$$

where all the components, with the exception of $\{\nu_s\}$ are of equivalent power with $\{\nu\}$ and the power $\{\nu_s\}$ is less than the power $\{\nu\}$.

Moreover, all the components of this sum are assumed as non- ssing.

Let us designate by $\Pi_{\{\nu\},\{\nu_j\}}$ the commutating operator, which converts the ensemble $\{\nu\}$ into and in the case of $\{\nu_s\}$ converts into $\{\nu_j\}$ an equiv part $\{\nu\}$.

Let us build up the operator $\tilde{P}$

$$\tilde{P} = E_{\{\nu\}} + E_{\{\nu_1\}}\Pi_{\{\nu\},\{\nu_1\}}E_{\{\nu\}}P + \cdots + E_{\{\nu_s\}}\Pi_{\{\nu\},\{\nu_s\}}E_{\{\nu_s\}}P$$

And the function $\tilde{f}$

$$\tilde{f} = E_{\{\nu\}}f_{(0)} + E_{\{\nu_1\}}\Pi_{\{\nu\},\{\nu_1\}}E_{\{\nu\}}f_1 + \cdots + E_{\{\nu_s\}}\Pi_{\{\nu\},\{\nu_s\}}E_{\{\nu\}}f$$

Then
$$\varphi_{(0)} = \tilde{P}^{-1} f$$

The synthesis problem may be stated for [illegible] implicit form $F_{s-1} - UF_s = 0$
The transfer into an explicit form would require the [illegible]mination of U^{-1}, which is of a complicated nature. possible to synthesize the automat for the opera[illegible] i.e., to determine $P = K_m \cdots K_1$ and then t[illegible] [illegible]er to $P^{-1} = K_1^{-1}, K_2^{-1}, \ldots, K_m^{-1}$, wh[illegible] [illegible]ves the problem. Here we, naturally, assume that the inverse operators K_j^{-1} exist and are known, since the [illegible] are known. The starting function in this [illegible] as in the synthesis of the automat for U .

In the above described statement of the synthesis, the automat reproduces the required states only in components, with numbers defined by the set $\{\nu\}$. While the states other components, which were regarded as auxiliary at t[illegible] synthesis of the automat for U, may have independent [illegible]es at the synthesis of the automat for another function. [illegible] other words, the synthesized automat may correspond [illegible] the part of the components $\{\nu_1\}$, to the problem for the operator U_1 with an initial state $f_{(0)}$, while in the other part $\{\nu_2\}$ it will correspond to the problem for the operator U_2 with the initial state $f_{(0)}$. The relation between U_2 and U in this case is determined by the equality:

$$\mathcal{E}_{\{\nu_1\}}U_1\mathcal{E}^{-1}_{\{\nu_1\}} = \mathcal{E}_{\{\nu_2\}}U_2\mathcal{E}^{-1}_{\{\nu_2\}}$$

The important particular case when $U_2 = U_1 = U$ may be realized, if $\mathcal{E}^{-1}_{\{\nu_2\}}\mathcal{E}_{\{\nu_1\}}$ may be commutated with U .

The questions discussed in this paper were directly related to the synthesis of automata. However, the same questions may be applied to the analysis of the automata. In particular may be determined the classes of operators, for the realization of which the given automat may be used. The problem of the operator P recovery may be also raised and in certain conditions solved on the basis the results of the operation of the automat if specially selected text functions are introduced into it.

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Above are detailed different methods of approaching the logical description of the dynamics of computer functioning, these computers being regarded as finite automata of a special type. The method of time logical functions allows to represent the microstructure of the circuit as well as the functioning of registers and systems of registers.

The description of the automat in terms of time logical functions allows to characterize the number of the circuit components, the complexity of feedback systems, and the dynamics of the operation of separate components. The presentation of the same automat by means of a set of recursive functions allows to connect more tightly its logical structure with the operator scheme of the algorithm.

Of the greatest interest in this connection is the prospective of finding the most effective structure of the computer control system ensuring the efficient realization of different classes of algorithms. A more detailed study of the processes occurring at each operational step of the computing machine may be made by investigating the command operators, described in the present paper. In its turn, the method of recursive functions allows to preset the order of execution of these operators, and the method of time logical functions provides for the presetting of their circuit execution.

All the afore-considered methods of formalization of a logical description reflect different sides of a unique problem - the synthesis of an efficient structure of automata on the basis of a description of its structure.

At the same time the usual approach to the theory of automata is so to say phenomenological, i.e. it reflects the system of transitions between states, but is isolated (alienated) from concrete realization.

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2

THE EXPERIENCE OF THE USE OF HIGH-SPEED COMPUTERS FOR SOLVING PARTIAL DIFFERENTIAL EQUATIONS

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1. The solving of partial differential equations is the most important problem of applied mathematics. Despite all the difference of the physical sense of problems solved by the help of modern high-speed computers, more than 90% of them are systems of equations in partial derivatives and ordinary differential equations. But while the methods of numerical solution of systems of ordinary differential equations are completely developed (except some questions which nowadays are not solved to the end and are mostly connected with the singular points of solutions), the intensive development of the methods of solving differential equations in partial derivatives began only after the creation of high-speed computers.

In "the premachine epoch" of mathematics there were constructed only some methods of solution for some narrow classes of partial differential equations, almost all of them linear. Only to the end of "the premachine epoch" such methods as the method of finite differences (in particular for solving linear elliptic equations) and the characteristic method (for the simpliest systems of non-linear equations of hyperbolic type), began to be worked out obtaining a certain degree of community. The high speed computers which in thousands and thousands times had increased the computing possibilities gave us an opportunity for carrying out the solution of such problems, which it was absolutely senseless even to discuss before.

Practical demands of scientific researches and technical planning put forward more and more complicated mathematical problems, and the development of computers goes behind the complication of mathematical problems raising in the course of scientific and technical progress. Therefore with the appeearance of high-speed computers, the problem of construction of new effective methods of numerical analysis becomes more actual and its significance increases more and more.

For the measure of the efficiency of the method we can take the time necessary for solving the problem with the practically necessary accuracy. The less the time is the more effective is the method.

While solving problems by the help of high-speed computers it is necessary to consider the time of the solution as the time spent by a mathematician for the development of a scheme of numerical solution, the times spent for constructing and debbugging of a programme, and the time of the calculation itself. The efficiency of the method of numerical solution of equations in partial derivatives is defined by the following factors:

1) the rapidity of the convergence of the method,

2) the community of the method,

3) the necessary accuracy of the solution,

4) the simplicity of the logical scheme of the solution,

5) the computer characteristics (the speed, the volume of the storage, the presence of the intermediate kinds of memory).

With the rare exception in cases when we can obtain an exact solution of an equation the numerical methods of solving equations in partial derivatives are approximate. Every method gives us usually an opportunity of obtaining different stages of the approximate solution, and we may consider an exact solution as a limit of a sequence of approximate solutions, while the number of the stage increases infinitely. For the rapidity of the convergence of the method one takes theoretically the rapidity of the convergence of the error of approximate solution to zero, while the number of the stage of the approximation increases infinitely. However, this theoretical rapidity of the convergence has only an indirect connection with the practical rapidity of the convergence.

We are satisfied practically with the finite accuracy of the calculation, restricting ourselves to some finite number of the approximation stage. And very often the limited laws of the convergence of the error to zero have not yet time to show themselves. Sometimes the method of solving can not be convergent at all, as, for instance, in the case of asymptotic methods, or even does not contain different stages of approximation.

At any rate, a mathematician , engaged in solving applied problems, always strives to construct a method, which would provide sufficient accuracy at any possibly small number of approximation stages. If there are now the ways for solving the problem of the estimation of the theoretical rapidity of the convengence, (although very difficult and not available for all methods used in prooves), then the estimation of the practical rapidity of the convergence is carried out until now only by the way of experimental calculations. The development of the practically rapidly convergent method itself is, I should say, some kind of art. where the author's intuition plays a large part. The practical rapidity of the convergence considerably depends upon the admissible value of an error. By the increase of the demands for the accuracy of calculation the practical rapidity of the convergence draws together with the theoretical one, and the estimation of efficiency of different methods may fully change itself with the change of the demands for the accuracy of calculation. If ,for example, the provided demands for the accuracy while solving Shrodinger's equation are not high, the methods of the theory of perturbations are very effective. But these methods are practically unapplicable in order to get the accuracy (7-9 decimal digits), which is obtained by the spectroscope measurements.

In the mathematician's researches one could always see two different tendencies. On the one hand-the mathematicians tried to create general methods suitable for solving a wide class of problems, on the other hand-they strove to solve in the best way the concrete problem given.

The generality of methods makes the task of a mathematician who starts solving a concrete problem, of course, easier and shortens the time spent for the development of a scheme of the numerical solution. However, this does not limit the significance of the community of methods. At present the methods, which would be appropriate for the equations with discontinuous coefficients or for obtaining discontinuous solutions attain special importance. The problems of such kind araise in the field of gas dynamics with the presence of shock waves, while studying heat and diffusion processes in heterogeneous medium or with the change in the process of aggregate state of matter.

A mere transference of the methods developed for the continuous processes upon the cases mentioned above may bring to wrong results, and therefore the generalization of the methods is principally necessary.

It would be wrong, however, to neglect the importance of particular methods and to direct all the mathematic researches for the development of general methods. It is natural that the particular methods specially developed for a narrow class of problems may be rather more efficient, than the general methods. If this type of problems has a great applied significance , the development of the special methods is quite proved.

While using modern high-speed computers the time of calculation on a computer is as a rule rather less than the time spent for the development of the scheme of the solution and for the construction of a programme. Therefore with the increase of the computer's speed we must prefer the methods with the simple logical scheme of the calculation,although the transference to more complicated logical schemes could significantly shorten the time of the caiculation . It is also necessary to notice that the automatization of programming still more strengthens the significance of the simplicity of the logical soheme.

II. The method of finite differences is nowadays perhaps the most general method for the numerical solution of partial differential equtions from the all known ones. This method first created for the linear equations of the elliptic type, was then widespread for the equations of hyperbolic and parabolic types, and is used successfully for the linear systems as well as for the non-linear ones.

There is much literature concerning this methods , the conditions of the convergence and the stability for the different types of equations are found out.

Although in the field of the non-linear equations the strict proof of the convergence does not always exist ,in practical calculations the convergence rarely gives rise to doubt. In the Soviet Union much attention was paid to the improvement of calculations in the method of finite differences. The demands of the convergence and stability of calculations in the applioation to hyperbolic and parabolic equations make us to give up the use of expilicit formulas by the transition from oue moment of time to the following one (here we speak about the time conditionally, other physical values may play the part of time, for example, one of space coordinates). But in the implicit formulas at each time intetval it is necessary to solve a system of linear algebraic equations.

In the problems practically solved nowadays such systems often contain several hundreds of unknown values. The application of the iteration or the elimination methods would make a perfect solution of a problem with such a large number of the mesh points of the net practically impossible.

The so-called "drive through" method proposed in 1952 by Gelfand I.M., Keldysh M.V. and Lokutsievsky O.V., helps to overcome the difficulties mentioned above. The idea of the method can be easily illustrated on a following equation

$$\left.\begin{array}{ll} \varepsilon u'' = p(x)u + f(x)\ ; & [\varepsilon > 0,\ p > 0] \\ \text{with boundary conditions} & \\ \text{at} \quad x = 0 & u' = \alpha_0 u + \beta_0 \\ \text{at} \quad x = \ell & u' = \gamma_\ell u + \delta_\ell \end{array}\right\} \quad (A)$$

If we shall consider this equation as a result of the replacement of time derivative by a finite difference expression, then ε will be a small value proportional to a certain degree of the time interval. the solution of ε boundary problem (A) one may bring to the solution of two Koshy's problems, for example, first finding the particular solution (U) which satisfies the condition at $x = 0$, then the solution of the homogeneous differential equation (u_1) with the homogeneous condition at the end and to obtain the necessary solution as the sum $U + Cu_1$, selecting the constant C ffrom the condition at $x = \ell$. However, at small ε both solutions U and u_1 will increase very rapidly and the necessary solution will be obtained as a small difference of large values. That is, that such a way of calculation will be unstable.

Thus the problem is to obtain a stable method of calculation at which we should directly obtain necessary solution. We can obtain it by the following way.

The necessary solution we shall obtain from the relation

$$u'(x) = \alpha(x)u(x) + \beta(x) \tag{B-1}$$

Then the differential equation (A) gives for $\alpha(x)$ and $\beta(x)$ the following equatons.

$$\begin{aligned} \varepsilon(\alpha' + \alpha^2) &= \rho(x) \\ \varepsilon(\beta' + \alpha\beta) &= f(x) \end{aligned} \tag{B-2}$$

Submitting α and β by such conditions

$$\alpha(0) = \alpha_0 \ , \quad \beta(0) = \beta_0 \tag{B-3}$$

we automatically satisfy the condition at $x=0$ for $u(x)$. It is important that the function $\alpha(x)$ and $\beta(x)$ increasses at the decrease of ε only as $1/\sqrt{\varepsilon}$ (and not as $\exp\frac{M}{\sqrt{\varepsilon}}$ for U and u_1).

From the relation (B-1) we further obtain the Koshy s conditions at the right end for the necessary solution of the boundary problem (A) .

$$u(\ell) = -\frac{\beta(\ell) - \delta_e}{\alpha(\ell) - \gamma_e} \ ; \qquad u'(\ell) = \frac{\alpha(\ell)\delta_e - \beta(\ell)\gamma_e}{\alpha(\ell) - \gamma_e} \tag{B-4}$$

The stability of calculation according to the equations (B-1) , (B-2) is provided at any positive α_0 and those negative α_0 , which satisfy the condition

$$|\alpha_0|\ell \ll \left(\sqrt{\frac{\ell}{\varepsilon}\int_0^\ell \rho(x)dx}\right)^{-1} \tag{B-5}$$

In the application to a finite-difference solution of partial differential equations this condition may be fulfilled at a small enough time interval.

If we shall substitute the differential equation (A) by a finite - difference expression, let us say, in the form

$$u_{K+1} - B_K u_K + C_K u_{K-1} = F_K \qquad (C-1)$$

then the "drive through" method will lead to a recurrent calculation of three functions.

Calculation from the right to the left for u_K :

$$u_K = \frac{\beta_{K+1}}{C_{K+1}} u_{K+1} + \frac{z_{K+1}}{C_{K+1}} \qquad (C-2)$$

Calculation from the left to the right :

$$\beta_{K+1} = \frac{C_{K+1}}{B_K - \beta_K} \; ; \quad z_{K+1} = \beta_{K+1}(z_K - F_K) \qquad (C-3)$$

For more complicated cases , when the system of equations in partial derivatives or the equation with two space coordinates is being solved ,the "drive through"method was generalized by Babenko K.J, and Chentsov N.N. (The matrix "drive throgh method"). Designating the system of the unknown values by means of two vectors $\bar{u}$ and $\bar{v}$, we shall write down a finite-difference scheme as following

$$\begin{aligned} a_{11}^m \bar{u}_{m+1} + a_{12}^m \bar{v}_{m+1} + b_{11}^m \bar{u}_m + b_{12}^m \bar{v}_m &= \bar{f}_1^m \\ a_{21}^m \bar{u}_{m+1} + a_{22}^m \bar{v}_{m+1} + b_{21}^m \bar{u}_m + b_{22}^m \bar{v}_m &= \bar{f}_2^m \end{aligned} \qquad (D-1)$$

where $m = 0, 1, 2, \ldots, M-1$ (M is a number of the mesh points of a net) a_{ij}^{m} , b_{ij}^{m} the given rectangular matrixes f_i^{m} the given vectors.

Let the system of boundary relations be represented in the terms

$$\bar{u}_0 = X_0 \bar{v}_0 + \bar{x}_0$$
$$\bar{v}_M = Y_M \bar{u}_M + \bar{y}_M \qquad \text{(D-2)}$$

where X_0 and Y_M - matrixes $\bar{x}_0$, $\bar{y}_M$ -vectors. We want to obtain the solution of the system (D-1) in such a form

$$\bar{u}_m = X_m \bar{v}_m + \bar{x}_m$$
$$\bar{v}_{m-1} = Z_m \bar{v}_m + \bar{z}_m \qquad \text{(D-3)}$$

The substitution of expression (D-3) in the initial system (D-1) yields reccurent relations for matrixes X_m and Z_m and vectors $\bar{x}_m$ and $\bar{z}_m$ by analogy with the simpliest case of the one-dimensional "drive through". However, certainly, the volume of calculation considerably increases. Besides that, the matrix "drive through method" contains more "underwater stones" than the one-dimensional one, although there are sufficient conditions here providing the stability of calculation (by analogy with the condition B-5).

3. As it was mentioned above in many important applied problems it is necessary to consider

equations with discontinuous coefficients or discontinuous solutions of equations. Physical reasons are often used in these cases for the construction of methods of solution. While calculating gas movement the shock waves appearing may be removed by the introduction of viscosity in the initial equations of gas dynamics. At the sufficiently small viscosity the solution of the equations will be continuous but in the vicinity of a shock wave the gradients of speeds and pressure will increase abruptly - the more the less the viscosity is.

Let us consider the simpliest wave equation

$$\frac{\partial u}{\partial t} + \frac{\partial F(u)}{\partial x} = 0 \qquad \text{(E-1)}$$

Viscosity may be introduced into the main differential equation

$$\frac{\partial u_\varepsilon}{\partial t} + \frac{\partial F(u_\varepsilon)}{\partial x} = \varepsilon \frac{\partial}{\partial x} \psi \left[\frac{\partial u_\varepsilon}{\partial x} \right] \qquad \text{(E-2)}$$

where ψ is a monotonous function of its argument, such that $\psi(0)=0$.

The solution of the equation (E-2) will be already continuous and ordinary stable methods may be applied to it. It is possible to show, that $u_\varepsilon(x,t) \to u(x,t)$ at $\varepsilon \to 0$ (if the initial conditions for u_ε and u coincide).

By the numerical solution of the equation (E-2) the coefficient of viscosity ε may be diminished together with the decrease of space and time intervals.

Replacing the equation (E-2) by a finite-difference relation

$$u_{m,n+1}-u_{m,n}+\frac{\tau}{2h}\left[F(u_{m+1,n})-F(u_{m-1,n})\right]=$$
$$=A_{m,n}\left(u_{m+1,n}-u_{m,n}\right)-A_{m-1,n}\left(u_{m,n}-u_{m-1,n}\right) \quad \text{(E-3)}$$

(here h is the net step according to space, τ - according to time, n is the number of the coordinate t, m are the numbers of the coordinates x) it is necessary to choose the coefficients $A_{m,n}$ in such a way as to approximate the operator (E-1) on smooth intervals of the solution by the operator (E-3) with the second order of accuracy.

In the neighborhood of the discontinuties the formulas (E-3) must be of the first order of accuracy. We may obtain this by submitting the coefficients $A_{m,n}$ on a definite dependence from their values of u in two neighboring points of a net $x=(m-1)h,\ mh$. For linear equations we may show the convergence of the method if the formula (E-3) provides a stable calculation. But the experience also confirm the convergence in the non linear case.

The given method is developed by K.I.Babenko and V.V.Rusanov for an equation of the type (E-1) as well as for systems of equations (in the last case u and $F(u)$ may be considered as vectors, and the coefficients $A_{m,n}$ as matrixes), and also for the equations with two space independent variables of the type

$$\frac{\partial u}{\partial t} + \frac{\partial F(u)}{\partial x} + \frac{\partial G(u)}{\partial y} = 0 \qquad \text{(E-4)}$$

(u, F and G n-dimensional vectors).

The method of introducing of the viscosity is not the only one (although it is apparently the most general) for obtaining discontinuous solutions. The direct obtaining of discontinuous solutions may be provided by the help of special construction of difference schemes, which would reflect integral laws of conservation on discontinuities. Thus, while solving problems of gas dynamics, these integral laws are the laws of conservation of mass, quantity of movement and energy.

If a finite-difference scheme with the step striving to zero, gives an exact relations on discontinuties , then the solution of finite-difference equations will tend to a necessary discontinuous solution.

For instance the system of gas dynamic equations

$$\frac{\partial u}{\partial t} + \frac{\partial p(E,v)}{\partial x} = 0$$

$$\frac{\partial v}{\partial t} - \frac{\partial u}{\partial x} = 0 \qquad \text{(F-1)}$$

$$\frac{\partial}{\partial t}\left(E + \frac{u^2}{2}\right) + \frac{\partial pu}{\partial x} = 0$$

correspond to integral relations

$$\oint u\,dx - p\,dt = 0$$

$$\oint v\,dx + u\,dt = 0 \qquad \text{(F-2)}$$

$$\oint \left(E + \frac{u^2}{2}\right)dx - pu\,dt = 0$$

One of the difference schemes, providing the solution of the integral relations (F-2) may be written down in the form

$$u_{m+1/2}(t+\tau) = u_{m+1/2}(t) - \frac{\tau}{h}\left(P_{m+1} - P_m\right),$$

$$v_{m+1/2}(t+\tau) = v_{m+1/2}(t) + \frac{\tau}{h}\left(U_{m+1} - U_m\right), \qquad \text{(F-3)}$$

$$\left(E + \frac{u^2}{2}\right)_{m+1/2}(t+\tau) = \left(E + \frac{u^2}{2}\right)_{m+1/2}(t) - \frac{\tau}{h}\left(P_{m+1}U_{m+1} - P_m U_m\right)$$

while the speed and pressure at the bounds of the layers U and P may be obtained from relations (exact or approximate) on discontinu - ties. This method was proposed by S.K.Godunov.

The works of A.N.Tickonov and A.A.Samarskii gave the most complete research of finite difference schemes in the application to discontinuous coefficients and solutions.

In these works the question about the conditions, which a homogeneous difference scheme has to satisfy, is put. This scheme provides convergence to the solution of a differential equation, for a possibly wider class of differential equations, in particular, admitting discontinuous coefficients and solutions.

For the homogeneity one may take here the constancy of the calculation scheme for the whole interval for all class of differential equations (that is the scheme must be the same, when the discontinuities take place, as well as at their absence.

The importance of this question may be illustrated by a following example. Let us take the simpliest heat conduction equation

$$\frac{d}{dx} k(x) \frac{du}{dx} = 0 \quad , \quad u(0) = 0 \quad , \quad u(1) = 1$$

Let

$$k(x) = \begin{cases} k_1 = const & x < \xi \ ; \\ k_2 = const & x > \xi \ ; \end{cases} \qquad \frac{k_2}{k_1} = \varkappa$$

At the point of discontinuty the following conditions must be fulfilled

$$u_{\xi-0} = u_{\xi+0} \qquad (ku)_{\xi-0} = (ku)_{\xi+0}$$

One may easily find the solution of this equation

$$u = \begin{cases} \dfrac{\varkappa x}{1+(\varkappa-1)\xi}, & x < \xi \\[2ex] \dfrac{(\varkappa-1)\xi + x}{1+(\varkappa-1)\xi}, & x > \xi \end{cases} \qquad \text{(I)}$$

Let us replace the differential equation by the finite-difference expression

$$k_i' \frac{u_{i+1} - 2u_i + u_{i-1}}{h^2} + \frac{k_{i+1} - k_{i-1}}{2h} \cdot \frac{u_{i+1} - u_{i-1}}{2h} = 0 \qquad \text{(I)}$$

In our case this equation is being easily solved, and for the limited value u we obtain the expression

$$u = \begin{cases} \dfrac{x}{\xi + \dfrac{(5-\varkappa)(3\varkappa+1)}{(3+\varkappa)(5\varkappa-1)}(1-\xi)}, & x < \xi \\[4ex] \dfrac{\xi + \dfrac{(5-\varkappa)(3\varkappa+1)}{(3+\varkappa)(5\varkappa-1)}(x-\xi)}{\xi + \dfrac{(5-\varkappa)(3\varkappa+1)}{(3+\varkappa)(5\varkappa-1)}(1-\xi)}, & x > \xi \end{cases} \qquad \text{(II)}$$

The solutions (I) and (II) coincide only at $\varkappa = 1$ that is with the continuous coefficients $k(x)$

Let us consider a differential equation

$$L^{(\kappa,q,f)}_{(u)} = \frac{d}{dx}\left[k(x)u'(x)\right] - q(x)u + f(x) = 0 \qquad \text{(G-1)}$$
$$u(0) = u_0\,, \quad u(1) = u_1\,, \quad \kappa(x) > 0\,, \quad q(x) \geqslant 0$$

in the class of piecewise-continuous and piecewise-smooth coefficients $Q(m_\kappa, m_q, m_f)$ with derivatives of the order m_κ, m_q, m_f accordingly.

A homogeneous difference scheme, corresponding to this equation, we shall write down in the form of

$$L_h^{(\kappa,q,f)} y_i = \frac{1}{h^2}\left[B_i^h(y_{i+1}-y_i) - A_i^h(y_i - y_{i-1}) + Q_i^h y_i + F_i^h\right] = 0 \qquad \text{(G-2)}$$

The coefficients A_i^h and B_i^h are some functionals of the coefficient $\kappa(x)$, Q_i^h of the coefficient $q(x)$, F_i^h - of the coefficient $f(x)$

$$A_i^h = A[\bar{\kappa}(s)]\,,\quad B_i^h = B[\bar{\kappa}(s)]\,,\quad Q_i^h = Q[\bar{q}(s)]\,,$$
$$F_i^h = F[\bar{f}(s)]\,,\quad \bar{\kappa}(s) = \kappa(x_i + sh)\,,\quad \bar{q}(s) = q(x_i + sh)\,, \qquad \text{(G-3)}$$
$$\bar{f}(s) = f(x_i + sh)$$

The following theorems are established

1. The homogeneous, canonic $\left(B_i^h = A_{i+1}^h\right)$ difference scheme of the type (G-2) in order to provide the second order of accuracy with relation to the step h in the class of

piecewise-continu   efficients $Q(m_\kappa, m_q, m_f)$ must have the following form

$$\mathcal{A}_i^h = 1\Big/\int_{-1}^{0}\frac{ds}{k(s)}\,,\quad Q_i^h = \int_{-1/2}^{1/2}\bar{q}(s)\,ds\,,\quad F_i^h = \int_{-1/2}^{1/2} f(s)\,ds \qquad (G-4)$$

$m_\kappa \geqslant 3$, $m_q \geqslant 2$, $m_f \geqslant 2$, then this c   s also sufficient; in other words, this di...rence is the best one.

However, it is not always possible to evaluate the coefficients (according to the formulas (G-4) exactly. In this case the functionals $\mathcal{A}_{i,\ldots}^h$ itself are evaluated with the use of some approximate formulas. The convergence will be provided if the conditions

$$\frac{B_n B_{n+1}}{k_n} - \frac{\mathcal{A}_n \mathcal{A}_{n+1}}{K_\Lambda} = \rho(h) \to 0$$

are satisfied. Here the index n corresponds to the point of the coefficients discontinuity $K(x)$ (that is the discontinuity is between x_n and x_{n+1}

$\rho(h)$ - zero-functional, K_n - the value to the right, K_Λ - the value to the left. Generally speaking, however, the second order of the accuracy of the difference scheme will not be provided in thia case.

The necessary conditions for the providing of the second order of accuracy demand a more serious analysis of the order of approximation of a differential operator by a difference one (that is, values $\psi_i^h = L_h(u_i) - (Lu)_i$

In particular, if we divide each interval $x_i - x_{i+1}$ on $\mathcal{L}$ parts and use quadrature formulas, in order to obtain the approximate evaluation of the integrals (G-4)

$$A_i = \sum \left(\frac{\lambda_i}{k_{n,j-1}} + \frac{\mu_i}{k_{n,j}} \right) h,$$

the the necessary condition is, that $\mathcal{L}$ should be of the same order, as $\mathcal{N}$ ($\mathcal{N}$ is the number of mesh points of the main net).

These results formulated for an ordinary differential equation are directly applicable to a partial differential equation

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x} k(x,t) \frac{\partial u}{\partial x} + f(x,t) \qquad \text{(G-5)}$$

in the rectangular field, if the discontinuties of functions $k(x,t)$ and $f(x,t)$ are fixed in space (the position of the discontinuty points does not depend upon time). A.A.Samarskii gene - ralized these results also for the case, when the discontinuties of the coefficients $k(x,t)$ and $f(x,t)$ displace in time. The convergence of an implicit difference scheme

$$\frac{y_i^{j+1} - y_i^j}{\tau} = \frac{1}{h^2} \Delta \left(A_i^{j+1} \nabla y_i^{j+1} \right) + F_i^{j+1} \qquad \text{(G-6)}$$

at

$$\mathcal{A}_i^{j+1} = 1/\frac{1}{h}\int_{x_{i-1}}^{x_i}\frac{dx}{\kappa(x,t_j)}\ , \quad \Gamma_i^{j+1} = \frac{1}{h\tau}\int_{t_j}^{t_{j+1}}\int_{x_{i-1/2}}^{x_{i+1/2}} f(x,t)\,dt\,dx \qquad (G-7)$$

if the time τ and space h steps satisfy the relation

$$\frac{h^2}{\tau} = \rho(h) \to 0 \qquad h \to 0,\ \tau \to 0$$

is proved.

The convergent difference schemes and iteration schemes for non-linear equations with discontinuous coefficients of the form

$$\frac{\partial}{\partial t} c(x,t)u = \frac{\partial}{\partial x}\left[\kappa(x,t)\frac{\partial\psi(u)}{\partial x}\right] + f(x,t,u) \qquad (G-8)$$

are also developed.

The given above results refer to the evaluation of the continuous solutions, which in the discontinuty points satisfy the equality of "heat streams" (that is $\kappa_n u_n' = \kappa_A u_A'$

The methods are also generalized for more general conditions of conjugation

$$[\rho u] = 0\ , \quad [z u_x - s u] = 0 \qquad (G-9)$$

We have to notice that the representation of differential equations in an integral form lies in the basis of the methods mentioned above. Thus, for example, the equation (G-1) may be written down in the form

$$w_{i-1/2} - w_{i+1/2} - \int_{x_{i-1/2}}^{x_{i+1/2}} q(x)u(x)dx + \int_{x_{i-1/2}}^{x_{i+1/2}} f(x)dx$$

where $w = -Ku$ are some streams. The expressions (G-4) are the natural approximate representations of the integrals with the discontinuous coefficients.

4. The advantage of the method of finite differences, while solving non-linear hyperbolic equations with discontinuties, lies in the simplicity of the logical scheme, but at the same time the accuracy of this method is less than those of the characteristic method.

When the position of the discontinuties is known a priori (exactly or approximately) the characteristic method is used successfully. This usually takes place in stationary problems of gas dynamics. We may express an opinion that the characteristic method is more efficient for solving stationary supersonic problems of gas dynamics while the method of the finite differences for non-stationary gas dynamic problems. The characteristic method was broadly used in the works of J.D.Shmyglevskii, O.N.Katskova, and others. While solving problems with the characteristic method some difficulties appear near the transition line (that is at the bound of the elliptic and hyperbolic regions) because the characteristics of different families cross under very small angles, and an abrupt change of unknown values takes place too. Therefore the movement from the transition line is better to perform with the help of series, instead of the characteristic method. In this case the calculations are simplified with the help of the transformation of equations to characteristic coordinates.

The characteristic method is well developed for two independent variables, but its application for solving space problems is connected with much larger difficulties. In the case of the space some different variants of the characteristic method may be used. V.V.Rusanov proposed a very visual scheme of the method. His method may be called the "tetranedron method".

A triangular net is constructed on the initial surface. Through each side of the triangle a plane tangent to the Mach cone proceeding from the middle of the side, is constructed. In such a way tetrahedrons are constructed, which tops form a new surface. The definition of all values in the tetrahedron tops is obtained from the relations on characteristic planes along the medians of tetrahedron facets. Except the complete space method of characteristics a mixed method is also used. This method is the combination of the characteristic and the finite difference methods. The idea of the method may be illustrated by the simpliest case of the wave equation

$$\frac{\partial^2 \varphi}{\partial x^2} = \frac{\partial^2 \varphi}{\partial y^2} + \frac{\partial^2 \varphi}{\partial z^2} \qquad \text{(H-1)}$$

Replacing one of derivatives in the right hand side by a finite-difference expression we obtain the system of equations in partial derivatives, for instance

$$\frac{\partial^2 \varphi_i}{\partial x^2} = \frac{\partial^2 \varphi_i}{\partial y^2} + \frac{\varphi_{i+1} - 2\varphi_i + \varphi_{i-1}}{h^2} \qquad \text{(H-2)}$$

this system is already solved with the characteristic method. V.V.Sychev applied such a method in solving problems of a supersonic flow past a body of revolution. The derivatives with respect to the meridian angle are replaced by a finite difference expression in the cylindric system of coordinates.

5. As it was mentioned above at present the methods of solving ordinary differential equations are fully developed now. In connection with the fact those methods of solving partial differential equations which converge them approximately to the systems of ordinary differential equations attain a large practical value. Since the solution of system of ordinary differential equations of high order (especially

boundary problems) also demands a great computing work, it is natural that mathematicians strove to obtain a good approximation already while substituting the equation in partial derivatives by the system of ordinary equations of a relatively low order.

As our experience shows the integral relation method in this sense is a successful one. The method may be formulated by the following way. Let there in the field $a \le x \le b$, $0 \le y \le \delta(x)$ the system of equations in partial derivatives of the "divergent" type be given

$$\frac{\partial}{\partial x} P_i(x,y;u_1,\dots,u_n) + \frac{\partial}{\partial y} Q_i(x,y;u_1,\dots,u_n) = \\ = F_i(x,y;u_1,\dots,u_n) \qquad i=1,2,\dots,n \tag{I-1}$$

Dividing the field into m stripes, with the length δ/m and integrating the system (I-1) with respect to y across each stripe, we shall obtain the system of integral relations

$$\frac{d}{dx}\int_{y_k}^{y_{k+1}} P_i\,dy - \frac{d\delta}{dx}\cdot\left(\frac{k+1}{m}P_{i,k+1} - \frac{k}{m}P_{i,k}\right) + Q_{i,k+1} - Q_{i,k} = \\ = \int_{y_k}^{y_{k+1}} F_i\,dy \tag{I-2}$$

Replacing each of the integrals by a certain interpolation expression

$$\int_{y_k}^{y_{k+1}} P_i \, dy = \delta(x) \cdot \sum A_{k,r} P_{i,r} \qquad \text{(I-3)}$$

(and by analogy for the integrals of F_i and substituting (I-3) in the integral relations (I-2) we obtain together with boundary conditions the systems of ordinary differential equations of the m -th approximation for the definition of the unknown functions $u_{j,k} = u_j(x, y_k)$

For the solution of this system we apply the worked out scheme of numerical calculation of ordinary differential equations.

The form of the boundary of the field $\delta(x)$ may be also unknown here (a complementary boundary condition making the problem definite should be given for it). In this case the boundary is simultaneously defined from the system of ordinary differential equations.

The integral relations method was applied for the solution of different problems of the elliptic, parabolic and mixed types (P.I.Chushkin, O.M.Belotserkovskii, O.N.Katskova). The application of this method for solving non-linear equations of a mixed type (of an elliptic type in one part of the field, of a hyperbolic type - in the other one) is of a special interest. In the theory of partial differential equations these problems are the most difficult. Though until now we are not able to obtain the theoretical proof of the convergence of the method in these complicated cases, practical calculations, however, convincingly show, that the convergence takes place.

Let us briefly consider the solution of problems concerned to eigenvalues. Different approximate methods lead to problems concerning to the evaluation of the eigenvalues of matrix. We must take matrixes of high order for obtaining sufficient accuracy (especially while solving many-dimensional problems of eigenvalues), so that it does not go in the computer's storage. A.A.Abramov and M.G.Neuhauz proposed a method, giving a more accurate definition of the eigenvalues at the transition from the matrix of a lower order to that of a higher one. Let us assume the initial symmetric matrix $\mathcal{A}$ in the form

$$\mathcal{A} = \begin{array}{|c|c|} \hline a & b \\ \hline b^* & c \\ \hline \end{array}$$

Let λ_1 be the minimum eigenvalue of the matrix $\mathcal{A}$. We shall find the minimum eigenvalue λ_0 of the matrix a $(a\bar{x}_0 = \lambda_0 \bar{x}_0)$ Taking λ_0 for the approximate value λ_1 we may find a more accurate definition of this approximation

$$\tilde{\lambda} = \lambda_0 + \frac{(\bar{y}_0, \beta^* \bar{x}_0)}{(\bar{x}_0, \bar{x}_0)(y_0, y_0)} \quad , \quad y_0 = (\lambda_c - c)^{-1} \beta^* x_0 \qquad \text{(J-1)}$$

In this case it is important, that

$$\lambda_1 \leq \tilde{\lambda} \leq \lambda_0$$

that is the formula (J-1) always provides a more accurate definition. This method is also used for obtaining the next eigenvalues.

At present we may say about the methods of numerical solution of partial differential equations on the whole, that the methods of solving equations with two independent variables are so developed, that we may obtain the solution of sufficient accuracy by the help of modern high-speed computers.

The difficulties of numerical calculation grow rather quickly with the increase of the number of independent variables.

We have enough experience in solving problems with three independent variables, and if we do not demand high accuracy of the solution, the calculations are practically possible.

At the same time a general not-stationary space problem is very seldom available for solution. Many mathematicians work hard work in this direction.

We express a hope, that the efforts of mathematicians developing efficient methods of solving and engeneers' efforts directed at the increase of speed, the volume of storage, the logical possibilities of computers will soon be crowned with success, and the most difficult many-dimensional problems of mathematical physics will be available for solution.

[redacted]

3

METHODS OF SPEEDING-UP THE OPERATION OF DIGITAL COMPUTERS

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INTRODUCTION

All the various methods of accelerating the execution of operations, considered as one of the means for speeding-up calculations, may be characterized by one common feature, that is, the applicability of such methods does not depend of the concrete contents of the program.

Speeding-up calculations is achieved by the accelerated execution of the elementary "bricks" of the program, i.e. the computing operations.

The present paper does not deal with the methods of acceleration based on definite program characteristics (for example, the selection of the command system, address, memory organization and the use of assembled computing systems).

The methods of acceleration of computing operations may be classified into two groups by the nature of these operations, that is:

1. Logical methods of speeding-up the main computing operations.

2. Methods of accelerated calculation of elementary functions.

Para.1. ON THE PRINCIPLES OF ACCELERATION OF THE EXECUTION OF OPERATIONS

Any computing operation executed by the machine may be dismembered into a certain sequence of simple actions. Let us designate every such simple action, effected at the entry of some control signal from the control device, by the term EMO-elementary machine operation. Though this concept is not quite precise, it may be useful in many cases.

Thus, a computing operation may be regarded as an aggregate of various elementary machine operations performed in a certain order.

Obviously, this aggregate is not determined only by the computing operation to be executed, for the list of elementary machine operations depends, as well, upon the adopted algorithm. However, for the majority of basic computing operations a long time since was developed and at present definitely accepted a "classic" system of used elementary machine operations. To these belong elementary operations of binary addition, shift, code transfer and so on.

The same may be said about the methods of dismembering the computing operations into sequences of components of elementary machine operations. Here, too, there are firmly established rules and recommendations.

However, the classic algorithms of the execution of computing operations are not the most efficient from the point of view of rapidity of machine actions and, in cases of especially high speed operation requirements, they can not be considered as optimum. Evidently, this conclusion being quite trustworthy, may be made beforehand, otherwise, would be very little verisimilar the following statements:

a) the set of classical elementary machine operations is the optimum;

b) any sequence of elementary machine operations at the execution of a computing operation is algorithmically the shortest, i.e. it can not be replaced by a sequence with a smaller number of members. It may be possible to set and solve the problem of determining all accelerated algorithms, proceeding from a sufficiently large class of possible elementary machine operations, but being formulated in this manner the problem would result extremely complex and practically hardly feasible to solve it.

Efficient algorithms may be determined much easier by an artificial way.

Indicated below, are some accelerated algorithms obtained in this manner.

From the previous analysis can be made a classification of the algorithmetic methods for the accelerated execution of computing operations.

1. The method of reducing the number of consecutively executed elementary machine operations by:

a) elimination of superfluous elementary machine operations;

b) simultaneous execution of elementary machine operations.

2. The method of introducing special elementary machine operations (special in the sense that they differ from classic operations).

There is a third method of speeding-up the execution of computing operations which differs essentially from the algorithmetic methods, i.e.:

3. The method of reducing the time of elementary machine operation by selecting an efficient logical structure of employed circuits.

Para.2. ALGORITHMETIC METHODS OF SPEEDING-UP OPERATIONS

Let us consider first the algorithms of accelerated execution of multiplication operations.

According to known statistical data about 50% of the machine time is spent on the execution of multiplication operations. Therefore, speeding up this operation is of great importance.

The difference between various methods of accelerated execution of multiplication operations lies in their machine algorithms.

There are electronic digital computers in which the multiplication is performed as a single elementary machine operation. However, more frequently the machine algorithm of the multiplication operation takes the form of alternating shifts over one digit and additions (digit by digit multiplication). The first method of multiplication gives satisfactory results as regards speed of operation, but requires a great number of equipment (in quadratic relation to the number of digits).

At the digit by digit multiplication may be used a combined (coincidence-type) adder as well as a counter-type adder.

Combined adders involve the use of a serial (or serial-parallel) digit transfer.

However, the stored sum of partial products may be easier formed in a counter-type adder. Therefore, we shall deal with machine algorithms of the multiplication operation employing adders of this kind.

Speeding-up by overlapping of elementary machine operations of addition and shift, is achieved in the simplest case by shifting the multiplicand code at the moment of the following addition in the adder.

The sum of partial products at multiplication, according to this method, remains unmoved.

It is interesting to note, that with this method of multiplication, if no doubled accuracy products are required ("2 n" digits), the adder and multiplicand register may be executed with " n " digits. For this, a ring shift of the multiplicand code should be made in the multiplicand register, and the adder should have a circuit of cycle carry from the highest to the lowest order. Moreover, prior to each transfer of the shifted multiplicand code on the adder, in the latter are to be cleared the memory cells corresponding to the highest order of the transfered code.

Wide use has found the multiplying circuit in which the shift of the partial product is effected in the adder during the addition EMO and does not involve additional operations of components.

An interesting alternative procedure of speeding-up of the multiplication by overlapping is the method of the "travelling wave". According to this method, in the process of multiplication, the addition of several partial products is accomplished simultaneously in the same adder. This method involves the use of a special counter-type adder, in which a new addition may be started from the lowest orders side before the previous addition (or even several precedent additions) in the higher orders has been completed.

This method of multiplication permits to store a sum of partial products at a maximum frequency, conditioned by the admissible frequency of component functioning.

The methods of speeding-up the multiplication operation by way of reducing the number of executed addition and shift EMO, are based on the elimination of addition steps at the multiplication by the multiplier digits, in the code "0" and on the elimination of the addition steps at the multiplication by multiplier digit groups in the code "I".

The reduction of the number of shift steps in the two aforementioned cases is effected by the introduction of elementary machine operations of group shifts over 2,4,8 digits and so on, or over an arbitrary number of digits designated by "K".

In case the digits of the multiplier y contain many "I" the multiplication may be made using the formula $x \cdot y = \overline{x \cdot \bar{y}} + x$ where the sign $\overline{}$ designates the EMO of conversion into the reverse code. Time is gained in this case because the code $\bar{y}$ contains a small number of "I".

Maximum speeding-up of the operation is achieved by the introduction into the arithmetic device of a special arrangement for shifting the multiplicand code over an arbitrary number of digits "K", during one shift step (see further) and by a special conversion of the multiplier code S, defining the minimum number of additions and subtractions necessary in the process of multiplication, according to the scheme.

$$S\left[0,\mathrm{IOOIIIOIII}\right] = 0,\mathrm{IOIOOO{-}IOO{-}I}\ .$$

The average number of EMO of addition - subtraction in this case, as shown by appropriate calculations, is equal to:

$$\frac{1}{8}\left[\frac{80 + 24\,(n-2)}{9}\right] - \frac{1}{2} + (-1)^{n} \cdot \frac{1}{18}\ ;$$

where n - is the number of digits.

It may be easily shown that by employing only classical EMO, a considerable speeding-up of operations can not be obtained. Indeed, any multiplication algorithm in this case is defined by a special identical conversion of S' digits of the multiplier y . Obviously, $SS' = S'$ if S' corresponds to the most efficient algorithm. Let us assume that Sy and $S'y$ do not coincide and consider the case when the higher digit

Sy is greater than the higher digit $S'y$. (The case when S and S' interchange their roles is also considered analogously). Then,

$$Sy - S'y = Sy - SS'y \geqslant 0{,}10\text{-}10\text{-}10\ldots\ldots - 0{,}010101\ldots\ldots > 0$$

which is impossible.

For the multiplication operation may be used the method of "carry remembering", first proposed by M. Nadler (Czechoslovakia), which involves a special EMO. This method takes advantage of a specific feature of the multiplication operation consisting in the fact that, at this operation all intermediate actions, preparing the result, are executed by the circuit without "conditional" transfers. Owing to this, it is possible to considerably reduce the time of the multiplication operation by introducing a special addition EMO with incompleted carry. The carries occurring at addition during this EMO are memorized in a special register.

The memorized carries are taken in account at every new addition and cleared at the end of the operation. A considerable acceleration of the multiplication operation may also be obtained, if the memorizing of the carries is effected not in every digit, but in several equally distanced points of the adder.

Let us now consider the algorithms of accelerated division. Usually, the quotient digit is defined by the feature of the direct or reverse (additional) code of the remainder. However, the remainder code, besides the aforementioned information, contains some additional data, which frequently allows to determine at once the group of quotient digits and, thus, reduce the number of elementary machine operations. The idea of this method is that when a remainder is formed with a sufficiently small or sufficiently big absolute value, the following digits of the quotient shall be obligatorily a group of identical digits (zeros or units).

Let us assume that the divisor q is normalized, i.e. contains "1" in the highest digit. Obviously, if the code of the positive remainder contains in its highest digits a "K" number of zeros, then, besides "1", in the quotient digits are to be recorded also K - 1 zeros. For obtaining the next remainder, it is sufficient to simply shift the initial remainder to the left over "K" digits and subtract the divisor from the obtained number.

The case with the negative remainder A_0 of a sufficiently small absolute value is symmetrical to the just considered. Assume, that in the higher digits of the remainder code there are K units. Demonstrate, that K-1 following remainders will be positive and, consequently, besides "0" in the next quotient digits should be recorded K-1 units.

Evidently, the i^{th} remainder of A_i is equal to $A_i = 2^i A_0 + q$ provided all previous remainders were positive.

From the study of the A_0 code, and taking into account the normalization of the divisor q, it may be concluded, that when $i \leq K-1$ all remainders A_i are positive, i.e., as required.

The last A_K remainder may be obtained by shifting the A_0 code over K digits to the left and adding the number thus obtained to the divisor q.

Let now A_0 be a positive remainder approaching closely enough to q (such cases are more rarely encountered). This fact may be found out in the machine by means of a simple circuit analyzing the higher digits of the divisor and remainder.

In this case it is necessary to build up the quantity $A_0' = A_0 - q$ and record in the quotient the K units contained in the higher digits of this quantity. The next remainder is obtained by shifting the A_0' code to the left over K digits and adding the divisor q.

The case when a negative remainder A_0 of a big absolute value is obtained, is symmetrical to the case just considered.

The average number of M quotient digits, obtained in one addition or subtraction EMO (taking into account only small values of remainders) is determined for the case of numbers with many digits in the following way:

$$M = \frac{1}{2} + \frac{1}{4} + \sum_{K=3}^{\infty} \frac{K-1}{2^K} = \frac{3}{2}.$$

It may be demonstrated, that the indicated division algorithm (taking into account only small remainders) is the most effective cycle algorithm, which realizes the method digit by digit, on condition that besides the divider register is used only one adding register and only classic EMO.

Let us assume that in the i^{th} cycle in the adder is obtained a certain quantity $A(z_{i-1}, q) = z_i$ defining the following group of digits of the quotient $B(z_i)$. (A and B are designations of certain algorithms). It may be easily ascertained that all the z_i are nothing else but remainders and, consequently, $B(z_i)$ is the maximum determinable group of the quotient digits. However, minding that of the divisor q is only known that it is normalized, the method for obtaining the digit group described before, exhausts all the possibilities; as was necessary to demonstrate.

The machine algorithm of the division operation with simultaneous shift of the divisor is usually not employed for the reason that it either results in a loss of division signs and less accuracy in division, or requires a divisor register and adder of double length.

However, by using a ring shift of the divisor it is possible to eliminate the effect of shifts on the duration of the division operation without increasing the equipment and without any loss in accuracy.

This method presupposes the utilization of reverse codes and the presence of a circuit of cycle carry in the adder. Apparently, the shift of the remainder code on the adder may be practically not made, considering that the place of the point is "moved" over 1 digit to the right.

Correspondingly, on the adder is to be transferred the divisor with a ring shift to the right. At every such transfer, the position of the point and, correspondingly the position of the sign digit are "shifted" in the adder over 1 digit to the right. The further actions are obvious.

As an example of the execution of the division operation with the utilization of special EMO, may be cited the method of M. Nadler, realized by means of the addition EMO with an incompleted carry. However, in some cases, it is even possible that the sign of the remainder, i.e., of the quotient digit will be determined incorrectly. If it is assumed that in each digit of the quotient there is also "-1", then, using this method, any error in any one of the digits may be corrected at the expense of the following digits.

The operation is completed by reducing the obtained quotient to the ordinary form by subtracting two codes, corresponding to the positive and negative units in the quotient digits.

Speeding-up of operations may be achieved in many cases by eliminating normalization and presenting the numbers in not normalized condition. Thus, without considerable loss in accuracy, it is possible at multiplication to have only one of the multipliers normalized, and, moreover, its normalization may be partially overlapped in time with memory access of the other multiplier. If the result of addition is to participate in the subsequent addition, its normalization to the left is also not obligatory.

A certain speeding-up may be achieved by the representation of negative numbers in the machine in the reverse code, on condition that besides the sign is introduced the code feature. In this case, at algebraic addition, no time is wasted in the adder for the conversion - it coincides with the transfer of the code into the adder (at multiplication - into the multiplicand or multiplier register). It is expedient to introduce the code feature for digits as well.

It seems expedient to increase in the machine the number of active computing devices, capable of conducting computing operations in parallel and separately from each other, and capable to ensure a wide direct exchange of information by interaction. The presence of several active computing devices permits to obtain a more effective execution of comflexes of operations, as well as separate arithmetic operations. Let us consider the possible procedures for the realization of certain operations in conditions of an increased number of components of the arithmetic device.

Assume that:

$\{T\}_i^{\tau}$ - is the condition of the device (adder-S, register -R) at the τ-cycle of the i-stage of the process.

$\{T\}^{\to p}$, $\{T\}^{\leftarrow p}$ -shifts, to the left and to the right respectively, over p digits.

A. Calculation of $u = a b^x$ with multiplication in a descending order of degrees 2^x.

If

$$b_j = \varepsilon_n 2^j + \varepsilon_{n-1} 2^{j-1} + \dots + \varepsilon_{n-j}\,; \qquad b = b_n \tag{1}$$

Then: $b_{k+1} = 2b_k + \varepsilon_{n-k-1}$ u $b^2_{k+1} = 2^2 b^2_k + \varepsilon_{n-k-1} 2^2 b_k + \varepsilon_{n-k-1}$

Take part S_b, S_{b^2}, R_a u R_b

α) $\varepsilon_{n-k-1} = 0$

I) $\{S_b\}^1_{n-k-1} = \{S_b\}^{\leftarrow 1}_{n-k-2}$; $\{S_{b^2}\}^1_{n-k-1} = \{S_{b^2}\}^{\leftarrow 2}_{n-k-2}$

β) $\varepsilon_{n-k-1} = 1$

I) $\{S_b\}^1_{n-k-1} = \{S_b\}_{n-k-2} + \{R_a\}^{\rightarrow 2}_{n-k-3}$; $\{S_{b^2}\}^1_{n-k-1} = \{S_{b^2}\}^{\leftarrow 1}_{n-k-2}$

2) $\{S_b\}^2_{n-k-1} = \{S_b\}^{1\leftarrow 1}_{n-k-1}$; $\{S_{b^2}\}^2_{n-k-1} = \{S_{b^2}\}^1_{n-k-1} + \{S_b\}^1_{n-k-1}$

3) $\{S_b\}^3_{n-k-1} = \{S_b\}^{2\leftarrow 1}_{n-k-1}$; $\{S_{b^2}\}^3_{n-k-1} = \{S_{b^2}\}^2_{n-k-1} + \{R_a\}^{\rightarrow 1}_{n-k-2}$

B) Calculation of $u = ab^2$ with multiplication in an ascending order of degrees 2^x.

Let us designate: $\tilde{b}_j = \varepsilon_j 2^j + \varepsilon_{j-1} 2^{j-1} + \cdots + \varepsilon_0$; $\tilde{b}_n = b$

$\bar{b}_j = 2^{j+2} \tilde{b}_j$

Then: $\bar{b}_{k+1} = 2\bar{b}_k + \varepsilon_{k+1} 2^{2(k+2)}$ u $\tilde{b}^2_{k+1} = \tilde{b}^2_k + \varepsilon_{k+1} \bar{b}_k + \varepsilon_{k+1} 2^{2(k+1)}$

α) $\varepsilon_j = 0$

I) $\{S_b\}^1_j = \{S_b\}^{\leftarrow 1}_{j-1}$; $\{R_a\}^1_j = \{R_a\}^{\leftarrow 2}_{j-1}$

β) $\varepsilon_j = 1$

I) $\{S_b\}^1_j = \{S_b\}^{\leftarrow 1}_{j-1}$; $\{S_{b^2}\}^1_j = \{S_{b^2}\}_{j-1} + \{S_b\}_{j-1}$

2) $\{S_b\}^2_j = \{S_b\}^1_j + \{R_a\}_{j-1}$; $\{S_{b^2}\}^2_j = \{S_{b^2}\}^1_j + \{R_a\}^{\rightarrow 2}_{j-1}$; $\{R_a\}_j = \{R_a\}^{\leftarrow 2}_{j-1}$

The schemes A and B require at each stage the knowledge of only one figure " b ". Therefore they may be adapted for any process in which " b " is determined digit by digit (for example $b = \frac{2}{d}$, then $q = a\left(\frac{2}{d}\right)^2$).

The schemes considered above, may be realized in decimal code arithmetic devices, provided certain modifications, are brought in the schemes for the reason that doubling may be effected in the decimal code adder. Thus, multiplication of "a" by "b" in the descending order of degrees of 2^x is based on the presentation

$$ab = \{[(a \cdot \varepsilon_n 2 + a\,\varepsilon_{n-1})2 + a\,\varepsilon_{n-2}]2 + \cdots + a\,\varepsilon_0$$

and is realized in the machine arithmetic device from S_p and R_a by the operation $\{S_p\}_j^1 = \{S_p\}_{j-1} + \varepsilon_j\{R_a\}$

and $\{S_p\}_j^2 = \{S_p\}^{1-1} = \{S_p\}_j^1 + \{S_p\}_j^1$ and in the ascending order in the machine arithmetic device from S_p and S_a by the operation

$$\{S_p\}_j = \{S_p\}_{j-1} + \varepsilon_j\{S_a\}_{j-1}, \qquad \{S_a\}_j = \{S_a\}_{j-1}^{-1}$$

The direct execution of this multiplication requires the determination of binary digits. In this case it is advisable to use the well known decomposition of a proper decimal fraction into a binary fraction by the overflow of the adder S at the operation S^{-1}, obtaining binary digits in the descending order of the degrees 2^x. Minding the greater efficiency of multiplication in the ascending order, it is expedient to use the number b' , dual in relation to "b" i.e.

$$b' = \varepsilon_0 2^n + \varepsilon_1 2^{n-1} + \cdots + \varepsilon_n$$

At division, binary digits of the quotient were obtained as a result of a corresponding trial and error procedure, and proceeding to the multiplication with these numbers, it is possible to form a quotient decimal code in the adder S_q (instead of R_q).

In the same manner may be modified the afore described schemes for execution in the decimal arithmetic devices.

Para. 3. METHODS FOR REDUCING THE TIME OF ELEMENTARY MACHINE OPERATIONS

An important condition for the accelerated execution of the addition EMO is that each component of the add circuit is of single-shot type.

Usually in add circuits with single-shot operation components, for the addition procedure are used memory registers of both addends.

As a result it is possible to eliminate one of the memory operations, connected with the number transfer, as practiced in computing impulse adders. When it is not desirable to utilize in the add circuit the memory register of the 2-d number, the single-shot operation may be achieved by means of a scheme in which the code, of the number stored in the adder, is defined in each digit by the position of two memory components (the combinations "00" and "11" correspond to the code "0", while the combinations "10" and "01" correspond to the code "1").

The functions from the digit and from the carry in this circuit are divided between different memory components.

No simple mechanical solution has yet been found for reducing the time of the carries.

Adding devices in which the average time of the addition EMO is reduced by strictly noting the moment when the carry is completed, or by introducing "by-pass circuits" in the through carry circuit, are, for the time being, of extremely complicated design.

The problem of group shift acceleration may be solved by means of a special shifter.

The shifter (see Fig.1) is a ferrite matrix in which the information is simultaneously recorded on all the ferrites of the given column, (each matrix column corresponds to a certain digit of the recorded information).

All the ferrites of each matrix line are transpierced by a common reading wire. Besides the recording and reading wires, all ferrites which enter into one matrix diagonal, are transpierced by common shift wires.

In this manner, when a reading signal is applied to a bus, the number shifted over a certain number of digits in the direct or reverse code is read out.

The number of apparatus in this shifter is not greater than in usual shifting registers, but its functional diagram is more simple.

Para.4. CIRCUIT EXECUTION ALGORITHMS FOR COMPUTING ELEMENTARY FUNCTIONS

The calculation of elementary functions should be included in the list of main machine operations adaptable for circuirt execution may be considered the algorithms for the calculation of function values, developed from the "digit by digit" algorithms in the following directions.

Direct Scheme. a) Specially selected trial and error codes are periodically generated by transmitters;

b) The result of the trial and error procedure defines the method of formation of quantities by the arithmetic device. These quantities are consecutive approximations to the value $f(x)$.

Reverse Scheme. The trial and error codes are consecutively formed by the machine arithmetic device;

b) The value $f(x)$ is formed on the basis of the trial and error results from specially selected codes which are periodically generated by the code transmitter.

As elementary machine operations for circuit execution of the calculation of the values $f(x)$ may be adopted:

a) Addition $a_i \pm \varepsilon_i b_i$,

b) addition with shift $a_i \pm \varepsilon_i b_i 2^{z_i}$

The last elementary machine operation may be frequently used at $a_i = b_i$

$$b') \quad a_i \pm \varepsilon_i a_i 2^{z_i} = a_i (1 \pm \varepsilon_i 2^{z_i})$$

Elementary machine operation

b' - multiplication by the numbers $(1 \pm \varepsilon_i 2^{z_i})$ $\varepsilon_i = 0, 1$

depending on the results of the trial and error procedure.

It should be noted that any number z may be represented with an accuracy up to 2^{-p} in the interval $(1,2) - z = \prod_0^p (1+\varepsilon_i 2^{-i})$ in the interval $(0,1) - z = \prod_0^p (1-\varepsilon_i 2^{-i})$

Let us consider the execution of certain elementary functions on the basis of elementary machine operations a, b, b'

For the function $e^{(\pm)x}$ we start from the presentation

$$y = e^{(\pm)x} = (1 \pm \varepsilon_1 2^{-1})(1 \pm \varepsilon_2 2^{-2}) \ldots (1 \pm \varepsilon_p 2^{-p}) \qquad (1)$$

The trial and error codes have the following form:

$$a_i = \ln (1 \pm 2^{-i}) \qquad (i = 1, 2, \ldots, p)$$

Using the recurrent relation

$$y_{j+1} = y_j \pm \varepsilon_{j+1} 2^{-(j+1)} y_j \qquad (2)$$
$$y_0 = 1$$

we obtain the circuit of a device comprising an adder, binary code registers and a shifter (see Fig.3).

Depending upon the results of the consecutive trial and error, we calculate y_{j+1} by the addition of y_j to $j+1$ shifted over y_j digit to the right. At $y_0 = C$ we obtain the values of function, $C e^{(\pm)x}$

By means of the reverse scheme, we can calculate the function $y = \ln x$.

Using the periodically generated numbers (2) and the recurrent relation

$$x_{j+1} = x_j + \varepsilon_{j+1} 2^{-(j+1)} x_j$$

we may determine ε_{j+1}

$$\varepsilon_{j+1} = \operatorname{sgn} (x - \tilde{x}_{j+1})$$

where $\tilde{x}_{j+1} = x_j + 2^{-(j+1)} x_j$.

Depending upon the defined ε_{j+1} it is determined whether the addend a_{j+1} participates or not in the formation of the quantity $\ln x$.

For calculating the values of the functions $y = tg\, x$ it is expedient to adopt the following presentation

$$\tilde{x}_j = \varepsilon_1 x_1 + \varepsilon_2 x_2 + \ldots + \varepsilon_j x_j \qquad (j = 1, 2, \ldots, p)$$
$$x = \tilde{x}_p$$

$$tg\, \tilde{x}_{j+1} = \frac{A_{j+1}}{B_{j+1}}$$
$$x_i = arc\, tg\, 2^{-i}$$

In th , for the values A_{j+1} and B_{j+1} the following recur lations valuable take place:

$$A_{j+1} = A_j + \varepsilon_{j+1} 2^{-(j+1)} B_j$$
$$B_{j+1} = B_j - \varepsilon_{j+1} 2^{-(j+1)} A_j \qquad (j = 1, 2, \dots, p) \qquad (3)$$
$$A_0 = 0; \quad B_0 = 1$$

(with $A_0 = A_1$, $B_0 = B$ we obtain $\frac{A_p}{B_p} = \frac{A + B\,tg\,x}{B - A\,tg\,x}$

The values x_i are trial and error codes, on the basis of which are determined the ε_{j+1}

$$\varepsilon_{j+1} = Sgn\,(x - \tilde{x}_j - x_{j+1})$$

Next, by adding the shift (at $\varepsilon_{j+1} = 1$) are calculated A_{j+1} and B_{j+1} (or at $\varepsilon_{j+1} = 0$, $A_{j+1} = A_j$, $B_{j+1} = B_j$) and so on up to $j = p$. For determining $tg\,x$ it is necessary to make the division $\frac{A_p}{B_p}$.

For calculating the values of hyperbolic functions may be applied a procedure similar to that described for the exponential function.

The calculation of the values of the function $y = arc\,tg\,x$ is made by solving the equation

$$\delta = A - B\,tg\,y = 0$$

The trial and error procedure is effected for determining ε_{j+1} by the sign of the value δ_{j+1} where $\delta_{j+1} = A_{j+1} - \bar{B}_{j+1}$

The values A_{j+1} and B_{j+1} are determined by (3) and $\bar{A}_{j+1}$ and $\bar{B}_{j+1}$ from relations similar to (3).

$$\bar{A}_{j+1} = \bar{A}_j + 2^{-(j+1)} \bar{B}_j$$
$$\bar{B}_{j+1} = \bar{B}_j - 2^{-(j+1)} \bar{A}_j$$
$$\bar{A}_0 = 0 \quad \bar{B}_0 = tg\,y$$

For carrying out calculations in decimal arithmetic devices, the before mentioned algorithms, must be so modified that any shift to the right is excluded.

For the function $y = e^x$ the recurrent relation (2) is replaced by

$$\bar{y}_{j+1} = 2^{j+1}\, \bar{y}_j + \varepsilon_{j+1}\, \bar{y}_j \qquad (2)$$
$$\bar{y}_0 = 2^{-\frac{p(p+1)}{2}}$$

where $y_p = y_p = e^x$

For calculating the values $y = \ln x$ it is necessary to use the recurrent relation similar to (2') i.e.

$$\tilde{x}_{j+1} = 2^{j+1}\tilde{x}_j + \varepsilon_{j+1}\tilde{x}_j$$
$$\tilde{x}_0 = 2^{-\frac{p(p+1)}{2}}$$

Then

$$\bar{\tilde{x}}_{j+1} = 2^{j+1}\tilde{x}_j + \tilde{x}_j$$

For detection of ε_{j+1} it is necessary to calculate

$$Sgn\,(x - \bar{x}_{j+1}) = Sgn\,(2^{\frac{(j+1)(j+2)}{2}} x - \tilde{x}_j)$$

When calculating the values of the function $y = tg\,x$ the relations (3) are replaced by

$$A_{j+1} = 2^{j+1} A_j + \varepsilon_{j+1} B_j \qquad (3')$$
$$B_{j+1} = 2^{j+1} B_j - \varepsilon_{j+1} A_j \qquad (j = 1, 2, \ldots, p)$$
$$A_0 = 0\,, \qquad B_0 = 1$$

Para.5. SPEEDING-UP OPERATIONS AT MICROPROGRAM CONTROL

The use of microprogram control in digital computers, presents many positive features and also speeds-up machine operation.

This is achieved by introducing in the external alphabet of the machine several symbols of accelerated algorithms, included in the executed program as a characteristic elementary link.

For other terms, this is achieved by forming new computing operations which are characteristic for the executed program and allows to make the most efficient use of the equipment. Such methods of speeding-up calculations are intermediate between the methods considered before and methods related with the concrete pecularities of the program.

Thus, the gain in time, in this case, is the difference between the time needed for executing the calculations according to standard programs composed of external alphabet operations for a usual machines and the time for executing the microprograms in the machine with microcontrol. This difference is mainly caused by the following:

1. Microprograms do not include such operations as control transfer and memory of certain command addresses for matching the main program with subprograms.

2. It is not necessary for many algorithms to bring intermediate results to a standard form, for example, rounding off and normalization may be omitted.

3. It is possible to match in time different operations, as for example, simultaneously with the addition performed in the arithmetic device adder it is sometimes possible to read out from the memory the digits for the following actions.

4. In some cases the specific features of the algorithm may be used to advantage.

As known, the reverse value $\frac{1}{X}$, may be computed by the iterative formula

$$y_{i+1} = y_i (2 - x y_i); \quad (i = 0, 1, \dots, i_K) \quad y_{i_K} \approx \frac{1}{X}$$

Let us replace X by $2\tilde{X}$:

$$y_{i+1} = 2 y_i (1 - \tilde{X} y_i)$$

In this form the formula becomes convenient in that all the numbers participating in the calculation according to this formula are not more than a unit, provided a limited interval of X modification is used and the initial approximation y_0 has been appropriately chosen. (The multiplication by 2 may be performed by a shift or addition of the number with itself). This allows to make the calculations with a fixed point. Moreover, no time has to be spent on the subtraction, as $1 - \tilde{X} y_i$ represents an additional code $\tilde{X} y_i$, which at all iterations may be replaced by a reverse code without loss in precision.

It is known that at calculations according to the indicated iteration formula the number of true signs y_i is doubled with each iteration. Thus, in y_i may be left $n \cdot 2^{-i_K + i}$ of higher digits and all others may be discarded. This may be used to advantage for reducing the time spent on multiplication by employing y_i as multipliers. The average number of additions in multiplications in this case will be equal to n. The initial approximation y_0 may be computed by the formulas of the type:

$$y_0 = ax + b$$
$$y_0 = a + (b-x)^2$$

where a and b are constants. In this case the time of multiplication may also be reduced at the account of the small number of digits. It may be considered that as far as speed is concerned, the method just considered may compete with different methods of division digit by digit, including the accelerated methods, as well. At present, when single-sided high-speed memories of large capacity on paper lists, have appeared, it has become possible, by increasing the number of constants, to pass from one approximation polynomial for the elementary function to a series of such polynomials at separate intervals.

Let us assume that the function value is to be determined at the interval $0 \leq x \leq 1$. We divide this interval in two equal ones by their length, and, in each of them, make an approximation of the function by the polynomial in the s^{th} degree. For each x the group $s+1$ of constants will be determined according to the q of the highest digits of the number . The lowest $n-q$ digits represent the difference Δx between x and the nearest lowest table value of the argument. The approximating polynomials are calculated by the Horner diagram with s multiplications

$$(\dots(a_s \Delta x + a_{s-1})\Delta x + a_{s-2})\dots + a_0$$

Obviously, in the polynomials, represented in this way, the coefficients $a_1, a_2, \dots, a_s$ as may be expressed without an exact number of digits. As the values $(\Delta x)^i$ after the point, have not less than q_i zeros, i.e., the number of significant digits which they contain is not more than $n-q$ consequently, the coefficients a_i may have not more than $n-q_i$ significant digits. The advantage of this method of polynomial representation appears at multiplication, owing to the fact that these values may be taken with an uncomplete number of significant digits.

It may be easily seen that the first multiplier contains on the average $\frac{1}{2}(n-sq)$ units, the second $\frac{1}{2}[n-q(s-1)]$ units and the last $\frac{1}{2}(n-q)$ units. Thanks to this, the calculation of the polynomial is considerably accelerated.

The analysis of the methods for calculating the elementary functions $\frac{1}{x}, \sqrt{x}, e^x, \ln x, \sin x, \operatorname{tg} x, \arcsin x, \operatorname{arctg} x$ shows that provided slight modifications and additions are brought in the usual arithmetic device it may be used for the execution of these methods. Thus, for computing $\sqrt{x}$ it is necessary to provide an output in the control device of the last digit of the order, and the possibility of high-speed recording and adding constants in the order adder.

Under these conditions, the speed of calculation of elementary functions in the arithmetic device controlled by a microprogram of an appropriate arrangement is increased by several times.

Thus, the control by microprogram ensures the possibility of making efficient use of all executive organs contained in the machine.

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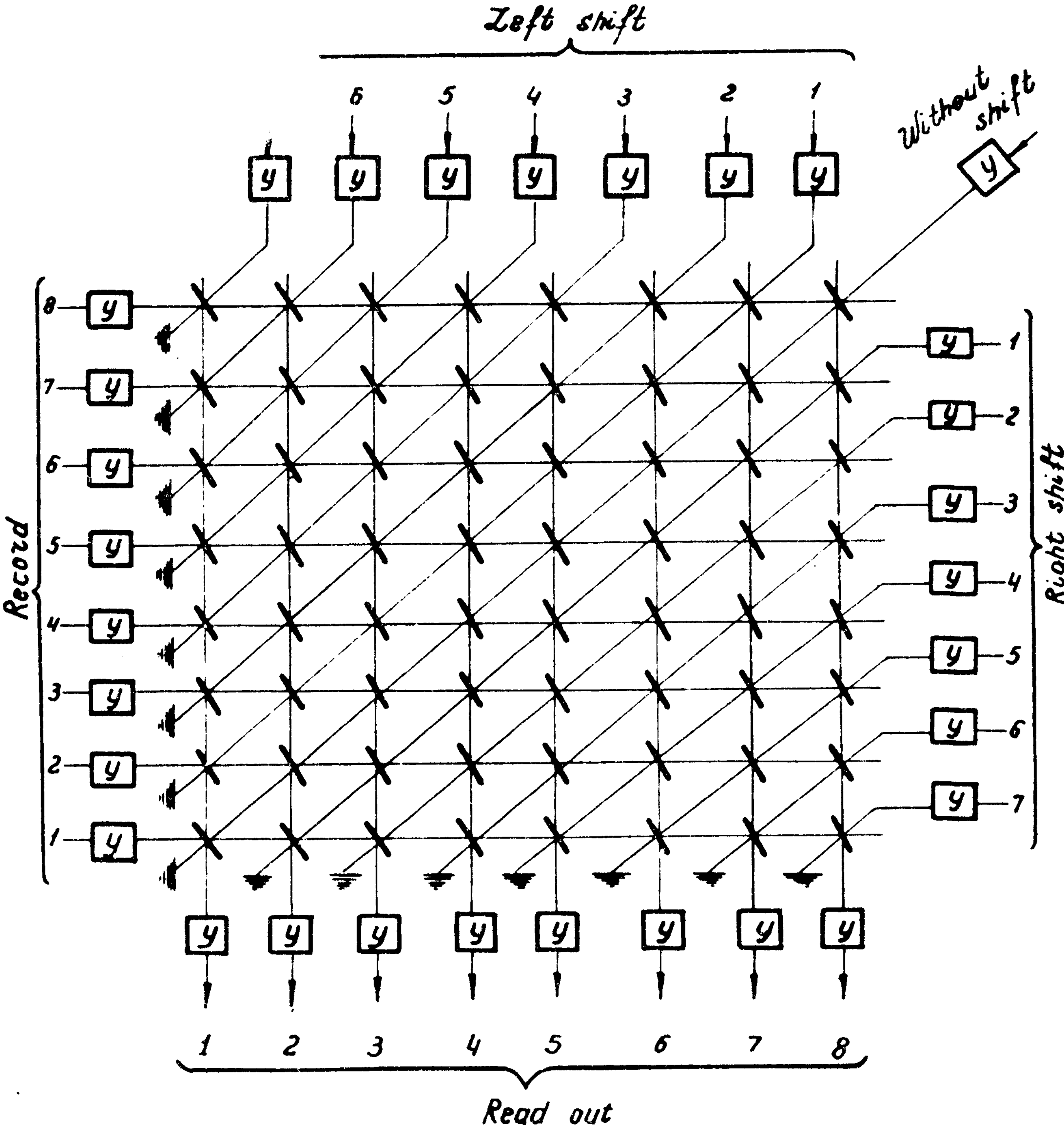

Fig. N1. Shifter.

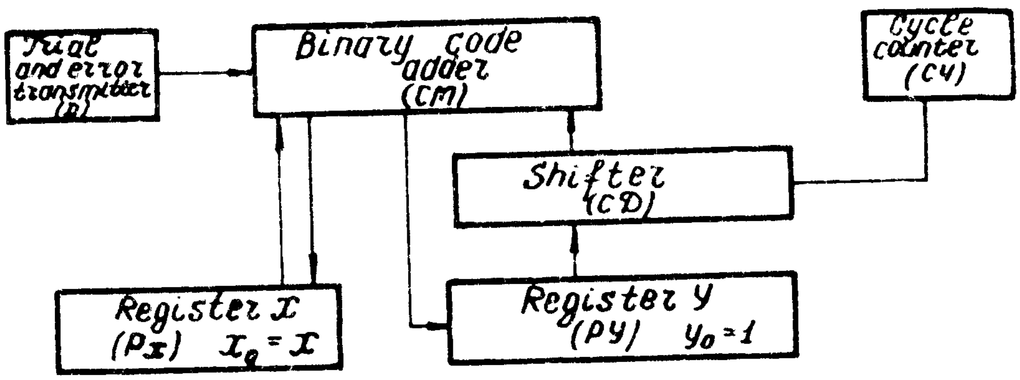
Trial and error transmitter
Binary code adder (CM)
Cycle counter (CY)
Shifter (CD)
Register X (PX) $X_0 = X$
Register Y (PY) $y_0 = 1$

Fig. №2

MACHINE TRANSLATION METHODS AND THEIR APPLICATION TO ANGLO-RUSSIAN SCHEME

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In this paper an account is given of a scientific research which has resulted in devising an algorithmic procedure for machine translation of different langu - ages into Russian /I/.
Methods evolved for translational purposes are explained, the Anglo-Russian scheme being chosen as an illustration of their application.

I. INTRODUCTION

Research in MT methods, which are outlined below,was started late in 1954 on the initiative of Academician A.S. Nesmejanov, President of the USSR Academy of Sciences. The first experiments in MT from English into Russian were carried out in December, 1955 /1,2/, which terminated the first stage of the research.

Some of the principles on which our research is based were put forward in earlier publications, among which a paper published in

RESEARCH, October 1957 /3/, can also be mentioned.

Since then considerable progress has been made towards adequate formulation of the method. We are now in the position to say, that the second stage of the research has recently been completed, in the course of which the suggested methods were shown to be of general applicability, for which purpose these methods were extended to cover MT from languages differing from English in structure as much as Japanese, Russian, Chinese and German /4/.

As to the Anglo-Russian scheme of MT the research here has reached a stage where complete grammatical analysis at a bilingual level as well as rearrangement of most important types of English idiomatic constructions can be accomplished, grammatical modification of the Russian translation (which indeed is the simpler part of the problem) being performed by an independent set of routines, termed Russian Synthesis.

In addition to this, the progress in Anglo-Russian MT has taken the form of considerable growth of the volume of words now entered into the MT dictionary. More than 2000 words are stored in the English section of our multilingual MT dictionary, a still greater number of Russian equivalents being stored in its Russian section. The dictionary thus is made to cover different fields of applied mathematics1/.

To complete this stage of research a large-scale test of the Anglo-Russian sheme has been carried out. 100 samples (which

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1/ Participants in this work were G.A. Tarasøova, whose contribution to the compilation of the Anglo-Russian Dictionary is most valuable, and L.M. Bykova.

amounted to 3000 sentences) of 'unknown text' were selected at random from different English authors, and translated into Russian in strict accordance with instructions provided by the MT dictionary and translational routines 1/. The ten persons chesen to carry out the experiment had no knowledge of English nor had they any previous experience with the tasks required 2/.

It emerged from the text that the scheme is very effective at dealing with all sorts of texts restricted, lexically, to applied mathematics, whereas grammatically no limitation as to type of the written text has been found necessary. 1 or 2 words per printed page is the average for 'unknown' words with the present size dictionary, which makes the translation quite adequate for understanding /See Tables Nos 1,2,3,4/.

For this reason as well as for reasons of preserving the proposed series of MT dictionaries strictly specialized as to field, we are not inclined to increase the volume of words in the present dictionary, but rather proceed with compiling medium size (say, 2500-3000 words each) dictionaries for various fields. This indeed will be our occupation at the next stage of research.

Translational routines for Anglo-Russian MT being final achievement of the recent research, it seems very reasonable, in the present communication, to lay particular stress on discription of translational routines for vocabulary and grammatical analysis of the English sentence. As to the principles on which MT vocabulary

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1/ A.I.Martynova was engaged as superviser in the testing procedure

2/ Several samples translated in this manner are given in Tables Nos. 1,2,3 and 4.

is based, the reader is referr.. to our earlier publications /3/.

2. GENERAL CONSIDERATIONS. APPLICABILITY OF MT METHODS.

Of two most general MT problems - those of possibility of machine translation and of its applicability - the former has already been resolved, both theoretically and practically, whereas the latter problem still remains open for discussion. The objective of the present research is to prove applicability of MT methods to any sphere of language.

To date, it is only within the limited sphere of scientific writing that the applicability of MT methods has won general recognition. As to other uses of MT, most machine translators are inclined to feel very doubtful /4/.

However, the majority of restrictions imposed on MT application, when analyzed, turn out to be due to a very strong inclination on the part of investigators to describe the translated language /sourse language/ in terms of correspondence to some other system, say, another language, or a group of languages, or science other than linguistics, especially logics or particular fields in mathematics. The possibilities of MT are discussed then as dependent on common elements in the compared systems. These elements may be more or less numerous, yet absence of complete correspondence between the systems, which is usually the case, inevitably brings about limitations to the scope of MT. Thus, application of machines to translating literary works of art has more than once been declared as absolutely ruled out (See, for instance, Ref.4. p.42).

In our opinion, it seems very reasonable to expect that these limitations can easily be eliminated, should the problem be formulated in a different way, namely, 'whether it is possible, within any language existing, to give formal description to any of its multiple spheres, individual as they may seem?'

This comes to the same thing as saying that the applicability of MT depends on whether it is possible to identify the implicit set of rules governing this or that particular sphere of language applications, be it as narrow a sphere as, say, Wordsworth's poetry, and, further, whether these rules can be formulated into a formal set.

Apparently, every piece of writing (insofar as written language is discussed) can be analyzed on these lines within the sphere where it belongs, and a set of rules for such analysis can be laid out. It is essential that these rules should be formal all along the line. Yet this is no obstacle either, since language is but a formal system of specific character developed by man to give communicative experession to his mental activities. As a consequence of the foregoing, it is immediately obvious, that problems posed by stylistic peculiarities of literary works of art can satisfactorily by resolved, if treated on the lines suggested above, i.e. within the sphere where they belong.

In this light, the supposed 'principal informalizability' of poetry (See Ref.4) should be rejected. Contrary to this supposition, poetry, as indeed any piece of literary art where formal elements are of no minor importance, is particularly susceptible to machine translation, in this sense.

This assumption has partly been justified on empirical grounds, that is, by experimental translation of passages from Ch.Dickens[1], J. Galsworthy, J. Aldridge and Edgar A. Poe. /See Tables Nos 2, 3 and 4/. It is our firm belief, that furter investigations will completely eliminate the restrictions imposed now on MT application.

An adequate description of a language, as indeed of any particular sphere of it, should finally aim at establishing within the analyzed system a set of correlations of the following type- $\boxed{\text{means} \rightleftarrows \text{effect}}$, by which the correlation of linguistic means and their meaning (effect) is inderstood.

Taken in its most general sense, the translational problem is, in effect, the problem of equating the aforesaid correlations of one language with those of another.

The procedure can symbolically be expressed by the following chart (See Fig.I.)

$$\begin{array}{ll} \boxed{\text{means}_1 \rightleftarrows \text{effect}_1} & \\ \phantom{\text{means}_1 \rightleftarrows} \ \ \| & \\ \phantom{\text{means}_1 \rightleftarrows} \boxed{\text{effect}_2 \rightleftarrows \text{means}_2}, & \end{array}$$

Fig.I.

where 'effect$_1$' and 'effect$_2$' are identical whereas 'means$_1$' and 'means$_2$' differ.

In the course of this substitution of one language for another, transposition of semantic content from one language to another is realized.

In conclusion, a final word must be added on the problem of MT prerequisites. These do not rest upon the existence of common basic elements in languages, as is often pointed out, but rather include the following two factors:

1/ Illustration to be found in Reference /3/.

I. language in itself is but a system of formal means by which communication of meaning is effected;

2. all language systems existing are so developed as to express in their particular ways any shade of meaning as well as various emotional effects.

When falling back on our symbolization, this comes to saying that the number of 'effects' in any two languages is equal, which makes the corresponding systems of 'means' fully comparable, through their 'effects'.

Since language systems are formal , any application of them can be provided with a description programmable on a machine.

3. A SHORT OUTLINE OF TRANSLATIONAL ROUTINES.

General procedure covered by translational routines can be broken down into three independent steps, these being:

I. Vocabulary Analysis of the source language for which purpose MT dictionary and a set of dictionary routines are used;

II. Grammatical Analysis of the source language for which purpose Analysis routines are devised;

III. Grammatical Synthesis of the target language for which ends the same set of Synthesis routines is applied to the texts translated from different source languages.

To make the outline concrete, the translational routines will further be described in their Anglo-Russian realization [1/].

A. Dictionary Analysis.

Dictionary Analysis of the English sentence starts with

1/ Complete list of translational routines given in the order of their application to be found in Table No 8.

searching every word of the text in MT dictionary. The first dictionary routine to be used here is that of transforming words of the text into the standard forms listed in MT dictionary (See Table No 5).

Thus 'wanted' will be transformed into 'want', 'stopped' into 'stop', 'coming' into 'come', 'lying' into 'lie','copies'into 'copy', 'bigger' into 'big', etc.

When dictionary search is completed, another routine is applied which concerns itself with the words that for various reasons have not been found in the dictionary. These are termed, 'unknown words', because their lexical equivalents remain unknown throughout the translational procedure. Yet, for the forthcoming grammatical Analysis, it is essential that grammatical qualification of the 'unknown words' should be obtained.

It is impossible to foresee every word in every text of a language or even of its particular sphere, since some of them may be occurring for the first time in the language, not to mention quite a number of more trivial reasons.

However, the 'unknown words' do not affect the translation, so far as grammatically they have been classified. To meet the latter problem,a very important routine, that of classifying 'unknown words' into 'parts of speech' has been devised,where extensive use in made of morphology and syntax of these words.

Another category of sentence constituents which undergo preliminary grammatical analysis in accordance with a dictionary routine, are the so-called 'formulas', by which various symbols used in different sciences are understood. Syntactical function of every 'formula' in the sentence is defined in accordance with a special routine.

So much for the words and symbols not found in MT dictionary.

In addition to lexical equivalents, words found in the dictionary are provided with information (termed 'invariant characteristics') which is partly grammatical partly semantic in character. For more detailed discription of this information the reader is referred to our earlier publication /3/. The only thing that need be montioned here, is that within the'invariant characteristics' obtained from the dictionary final and preliminary information is distinguished. Information is considered final for dictionary cycle when lexical equivalent of the word is included. Instead,preliminary information of the word is restricted to the indication 'homonymous' or 'polysemantic'.

Special routines have been devised to deal with homonymous and polysemantic words, the analysis of former words preceding that of the latter.

The four types of 'Homonyms' analyzed by the routine, are those of 'abjective-noun' (Homonym I),'noun-verb' (Homonym 2), 'verb-adjective' (Homonym3) and of 'preposition-adverb' (Homonym 4) Among Homonyms I, a more complicated sub-type is distinguished,- that of 'adjective-noun-verb' (Homonym I Complicated). See fig. 2

Cp.: CHECK:	I. adjective	-	контрольный
	2. noun	-	контроль
	3. verb	-	'polysemantic'
SQUARE:	I. adjective	-	квадратный
	2. noun	-	'polysemantic';
	3. verb	-	возвести в квадрат

Fig.2.

In specifying Homonyms 1, 2 and 3 a combination of morphological and syntactical analysis of the word is used. Thus, any inflection (except for ER or EST) identified in Homonyms 1 or 3 makes 'adjective' an impossible alternative, just as ED or ING inflexion in Homonym 2 cross out 'noun' solution. These are morphological criteria, which do not, however, find as wide an application as syntactical analysis does in view of scarce inflections in English.

The information Homonyms acquired in the course of this analysis may or may not be final for the dictionary cycle, since some of them are provided here with the indication 'polysemantic' instead of lexical equivalent(See Fig.2.)

Total number of polysemantic words stored in our dictionary amounts to 500 words.

Determination of multiple meaning is performed by specifying typical contexts of polysemantic words in acordance with a special routine which concludes Dictionary Analysis of the English sentence

Since basic principles of this routine have been discussed elsewhere (See Ref.3), we do not propose to dwell on them here. However, to make this paper comprehensible on its own, two illustrations of context analysis of polysemantic words have been included (See Tables Nos. 6 and 7). For details and the background of the method, the reader is referred to our earlier publication (3).

B. Grammatical Analysis.

Grammatical processing of a sentence is broken up into two independent steps, these being Analysis and Synthesis. The latter is the simplest of the two, for which reason it is not here that the main interest of the problem lies. So far, the discussion of Synthesis will be restricted to a few general comments.

Synthesis routines provide rules for grammatical modification of the translated text in accordance with grammatical information obtained in the course of the Analysis of the English original.

The most important peculiarity of Synthesis routines is their non-comparative nature, which means that rules of word-changing,as well as certain rules of word-building, are formulated strictly within particular target language. Owing to this, the same Synthesis routines can be applied to sentences translated from different languages.

However, Synthesis requirements are inclined to increase in case these routines serve multi-lingual MT purposes. With multi-lingual MT in view, Synthesis routines should be:

I. EXAUSTIVE in describing target-language word-changing system, since grammatical rules with no application in MT from one language may become vitally important when the source langauge is changed;

2. INFAILING in carrying through <u>any</u> instruction obtained from the Analysis of the source language, which makes necessary providing every 'non-productive' category of the target language with a 'productive' grammatical equivalent.

   So far, the problem of grammatical equivalents within a language, theoretically, stands out as most important for Synthesis in MT.

3. Synthesis routines should be INDEPENDENT of Analysis, since the latter may be very different for different languages.

Unlike Synthesis, 'independent' Analysis cannot be recommended, for this would not at all help to make it economical. Analysis problems being numerous and important scientifically, they indeed deserve special discussion which is given below.

English Analysis is covered by six routines, which are applied in the order indicated in Table No 8. In view of lenghth limitations of the present communication the discussion is restricted to general outline of most important Analysis routines, among which 'Verb' and 'Syntax' stand out as routines playing the key part in the whole precedure of Analysis.

B.I The 'Verb Analysis' routine is diveded into five sections, the first section being compulsory for every verb of the sentence, whereas of remaining sections only one is employed for each type of the analyzed verbs.

In Section I verb selection for further analysis is performed. Among words picked out for analysis in this routine are those possessing the indication 'Verb', so far as they do not have any of the following indications: 'to be Disregarded (D)','Not to be Changed' (NCh), or (Russian) indications 'Participle', 'Verbal Adverb' or 'Verbal Noun'. Check-up for absence of these indications is meant to exclude from further analysis those of the verbs that have been elsewhere provided with characteristics that satisfies Synthesis routines.

In addition to verb selection, correction of certain verb indications is envisaged in Section I.

Among verb indications liable to correction are those of tense with verb-predicates in if-clauses and of case government with link-verbs, as well as some more particular indications. Analysis of homonymous forms, such as Past Indefinite and Subjunctive, of irregular verbs also belongs here.

Check-u r grammatical context implying correction as possibility performed both when one of the above-mentioned indications is ascribed to the analyzed verb in the dictionary and when it is about to be developed in the course of further Analysis.

Preliminary Check-ups of Section I are followed by verb analysis proper, for which purpose the analyzed verb is sent to one of the four different sections, differences in morphological structure of the verb being decisive in choosing the section. Thus, verbs with S-eding[1/] are sent to Section II, verbs with ED-ending, as well as certain forms of irregular verbs, enter Section III, verbs with ING-ending are directed to Section IV, whereas verbs not inflected are analyzed in Section V.

Grammatical qualification of 'S-verbs' in Section II depends on whether S stand out as the only ending of the verb, or another ending (usually ING) is associated with it. In the latter case, the following indications are developed for the Russian equivalent verb: 'Verbal Noun; Neuter; Plural', which imply further analysis by the 'Noun' routine at the proper time.

1/ The term ENDING is applied to affixes following the stem of the word, whereas affixes preceding the stem are called PREFIXES.

N o t e, that the term AFFIX is restricted to those formatives that are used in word-changing, whereas, formatives used for derivation are termed SUFFIXES.

When S is the only ending, English characteristics of the verb (Predicate in the Present Indefinite form) is transformed into Russian indications, but not without checking-up for correction conditions (See above). Resultant characteristics is 'Predicate', associated with either 'Present' or 'Future tense', Number and Person (or gender for the Past tense in other cases) of the Russian predicate remain not defined until the subject of the Russin sentence is determined.

The analysis of 'ED-verbs', i.e. verbs with ED-ending and certain groups of irregular verbs, is performed in Section III, where syntax definitely takes precedence. The four main patterns of grammatical verb context analyzed here are indicated in Table No 9 as

Patterns I: Ia, Ib, Ic, Id;
2: 2a;
3: 3a, 3b;
4: 4a, 4b, 4c.

Noteworthy is the fact, that contex analysis of a word implies, in all cases, observation of 'Rules of Word Selection'. These rules are based on classifying all the words in a sentence into three categories, which are:

I. words of third-degree structural significance, where particles, adverbials, parentheses and coordinated[1/] parts of the sentence are included;

II. words of second-degree structural significance, where different words and word groups belong, so far as they are placed in the attributive position towards some word of a sentence;

1/ The term COORDINATED is applied to those parts of the sentence, which are introduced by a coordinating conjunction or panctuation mark.

III. words of first-degree structural significance, which include words not identified as belonging to either of the two previous categories.

Through application of 'Rules of Word Selection' in the course of searching procedure all words of lesser category than the word searched are omitted, chief constituents of the grammatical pattern required being thus singled out.

This is not the place to give a detailed discription of all the processes involved in the analysis of verb patterns in Section III. For this reason, the discussion will be restricted to just a few comments on patterns that bring about the most interesting solutions. They are Patterns I:Ia, Ib; and 2:2a.

Among different solutions of Pattern I:Ia noteworthy is that of transforming English construction of Modal Passive,

i.e.	Modal Verb	+	Selected Verb indicated 'Auxiliary I' (BE)	+	Analyzed Verb	,

into Russian Active Compound predicate.

	Modal		Analyzed
i.e.	Verb;	+	Verb,
	Impersonal		Infinitive,

the transformation being associated with conversion of English subject into Russian Direct Object (See Tables Nos.I0 and I2).

Pattern 2:2a is provided, among other solutions, with that of transformation of English Complex Object Construction into Russian subordinate clause.

Resultant characteristics developed for the verbs analyzed in Section III include both morphological and syntactical information.

Of syntactical indications only 'Predicate' and 'Attribute' are fixed here, the former being associated with morphological indications of mood (Indicative, Subjunctive and Infinitive are develop - ed here) of tense (Present or Past) and voice (both Active and Passive are here developed).

The indication 'Attribute' is accompanied by morphological indications of 'Participle', tense(Present or Past) and voice (Active or Passive).

ING-forms of the verbs are defined is Section IV, where the same verb patterns are analyzed, though important changes in their value affect the order in which they are searched here.

Pattern I:Ib disappears, whereas Patterns I:Id and 3:3a, 3b, are much wider represented here, the former pattern being compli - cated by differentiating quite a number of semantic groups of verbs significant for the Analysis. These groups are Nos I to II, class I, and Nos 2,7,9,21 and 24, class II (See below).

There are some differences in modifications of Patterns 4:4a, 4b. To complete the picture, another two patterns should be men - tioned, which are here introduced. They are Patterns 3:3c and 5:5a

The resultant characteristics of the Russian equivalent verb include one of the following sets of indications: I) 'Verbal Noun, Neuter'; 2) Participle, Present tense, Active voice; Attribute'; 3) 'Verbal Adverb, Present (or Past) tense; 4) 'Not to be Translated (NT), to be Disregarded' (D). In addition to these, 'Infinitive', 'Subjunctive' or 'Indicative mood', with the corresponding set in indications, is developed in case the analyzed verb takes it. characteristics from some of the 'selected (helping words' .

Verbs not inflected are analyzed in Section V. Verb Patterns I:Ia, Ib, Ic, Id are replaced here by I:Ie, If, Ig; Patterns 2:2b, 2c are added to Pattern 2:2a, which, in its turn, is greatly enlarged. Patterns 4:4a and 5:5a disappear; instead, Pattern 4:4c increases considerably, and Patterns 5:5b-I, 5b-2, 5b-3 are introduced.

Among different solutions of Patterns 5:5b-I and 5:5b-2 at least those two are worth special mention, which deal with transformation of the English constructions of Complex Subject and of attributive Infinitive into the Russian complex sentence or subordinate clause, accordingly.

The resultant information here includes the indication of Infinitive, Imperative, Subjunctive or Indicative mood, with the indications of tense (Present, Past or Future) and voice (Active or Passive) attached in case of the Indicative mood. The only syntactical indication fixed here is 'Predicate'.

Verb analysis in different section of the routine is based on the classification of the verbs, devised to characterize English verbs both within the English system and with regard to the Russian translational traditions.

Within the English language verbs are classified into 'MODAL' and 'HALF-MODAL (help, dare), AUXILIARY and 7 sub-classes of HALF - AUXILIARIes, CAUSATIVE (cause, enable, make, order, command, etc.), DECLARATIVE (declare, call, label, report, etc.), Verbs takin two Objects (give, offer, permit, etc), aso.

To meet the requirements of the Russian translational traditions, verbs are divided into classes and semantic groups. To date, 53 groups of verbs have been established.. These are summarized into three classes, the first two classes comprising verbs having translational peculiarities in finite (class I) or infinite (class II) forms; class III covers more complicated cases.

The 'Verb Analysis' routine is applied until every verb of the sentence is provided with all the grammatical information required in the Synthesis routines, except for the indications of number, person (or gender) which are not defined until the subject of the Russian sentence is established.

Noteworthy is the fact that the information obtained in this routine is not restricted to the analyzed verb, but is extended to cover the information,available at this stage of Analysis,concerning 'selected' (helping) words (verb, nouns, adjectives, aso.) and punctuation marks. Moreover, quite a number of transformations in sentence structure are introduced here, which include change of word-order, inserting necessary conjuncions and other words or punctuation marks, etc. These are transformations associated with the translation of Complex Subject and Complex Object, Attributive Infinitive and Gerundival Subject, as well as some other verb constructions.

B.2. The 'Verb Analysis' routine is followed by the routine devised to analyze the punctuation marks of the English text with the exception of those terminating the utterance[1/].

1/ Note, that our application of the term UTTERANCE is at variance with its usual applications. A piece of text, teminated by full-stops, exclamatory or question marks, we call 'utterance', in order to distinguish it from the SENTENCE, by which only a simple sentence is understood, i.e. a sentence containing not more than one non-coordinated predicate.

The analysis here serves two ends. One is to establish the 'English function' (i.e. a function within the English text) of every punctuation mark, the second aim being their 'Russian function', by which their Russian correspondents are understood.These functions may or may not coincide. In the latter case, both English and Russian indications are developed. Thus, commas marking out a prepositional phrrase, obtain English indications CP (Comma, Parenthetical), associated with Russian indications CD_R (Comma,to be Disregarded in Russian), as a result of which this comma will not appear in the Russian text.

There are cases, when English punctuation marks should be neglected in the course of English Analysis, though rendered by the same mark in the Russian text. This is achieved by developing theindication CD (Comma, to be Disregarded in English).

B.3. The 'Syntax Analysis' routines succeeds the Analysis of Punctuation Marks, since it is essential that the information which can be obtained in both previous routines should be available here. The analysis is carried out by three cycles.

In Cycle I, Parentheses, comparative AS-phrases and Attributive word-groups, with a participle, verbal adverb or adjective as chief constituent, are marked out by means of appropriate qualification (and insertion, when necessary) of punctuation marks. This qualification includes the development of indication $CP^{b/e}$ (Comma, Parenthetical, beginning/end) and $CA^{b/e}$ (Comma, Attributive, beginning/end).

Attributive groups are not isolated until preliminary check-ups for certain patterns of grammatical context have been carried out. Among these patterns are the following three (See Fig.3):

I. Preceeding word is CA^b

2. Preceeding word is Noun .... (on condition that it is associated ith Pattern 3)

3. Following word is
- 3a. Preposition;
- 3b. Conjunction /or Conjunctive Word/;
- 3c Punctuation Mark (PM);
- 3d Verb indicated 'Participle, Short form'.

(may or may not be associated with Pattern 2).

Fig.3.

Practically, isolation of the above-mentioned word-groups comes to establishing their right boarder ('end'), since the left boarder ('beginning') in these cases can easily be associated with the chief constituent of the construction.

The 'end' of the isolated word-group is searched to the right of the chief constituent until the nearest following

a) CA^e

or b) Noun with indication (or conditions of)'Subject';

or c) VERB with indication 'Predicate';

or d) Conjunction without indication 'coordinating';

or e) Punctuation Mark without indications CP or 'b' ('beginning'),-

is found. It is essential that the search should be performed in the order indicated above [1/].

1/ Mind that wherever following or preceding words are searched, 'Rules of Word Selection' are strictly observed in the Analysis routines.

In Cycle II sentence boarders are established by checking up the utterance[1/] for the presence of

I) Conjunctions with indication 'Inhomogeneous'(CI);

and/or.....2) Conjunctive words (nouns or adjectives);

" .....3) words with indication 'Initial';

" .....4) two (or more) Predicates within a passage terminated by sentence boarders already established;

a) immediately following each other,

b) not immediately following each other;

" ......5) two nouns following each other, but not joining in a 'lawful' combination.

A very detailed analysis of every pattern is carried out in the order indicated above. Ordinarily, sentence boarders do not acquire the indications SB (Sentence Beginning) or SE (Sentence End) at this stage of the analysis, but for two cases:

a) when pattern analyzed is

CI		Adjective		/Absence	
'of condition'	+	indicated	+	of	;
		'Predicative'		Noun /	

b) when two conjunctions follow each other, the latter being provided with a correlative conjunction or conjunctive word.

At this stage of the Analysis certain changes in the structure of the Russian text are also provided. In this conneetion, mention should be made of the insertion of the conjunctive word 'КОТОРЫЙ (with appropriate indications) in case it is omitted in the English attributive clause (Patterns 4 and 5).

In Cycle III information obtained by this time is used to qualify sentence-boarders as indicating 'Beginning' or 'End' of the

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1/ See above our definition of the term.

sentence inserted within the boarders of another sentence. Among other more particular cases, that of the sentences where subject and predicate are separated by an attributive or other subordinate clause, is analyzed here. Initial, middle or final possition of the non-predicative piece of the broken sentence is considered decisive for the order in which they are dealt with.

Boarders of the sentences which have not been recognized as 'insertions' within other sentences are qualified as SD (Sentence Division), since neither 'Beginning' nor 'End' indication is considered necessary here.

The information obtained by application of the 'Syntax Analysis' routine is extremely valuable, as long as syntactical units for further Analysis are marked out. Nouns, Numerals and Adjectives are analized within these units, the order in which Syntactical units are treated being as indicated below;

I. Sentence (minus all Parenthetical and Comparative or Attributive word-groups);

2. Comparative and Attributive word-groups (minus Parentherical word-groups) within this sentence;

3. Parenthetical word-groups within this sentence;

4. Next Sentence (minus all Parenthetical and Comparative word-groups);

Step 4 is again followed by Steps 2,3,4 aso., till the last sentence is looked through.

B.4. The 'Noun Analysis' routine is devised so as to cover the analysis of two word-classes, which are Nouns and Numerals. The so-called 'ordinal numerals' being qualified as Adjectives/5/, only cardinal numerals are termed Numerals here. These are not entered into the class of Nouns owing to their morphological peculiarities.

The routine is divided into two parts, the development of CASE indication being the target of Part I, whereas in Part II the indication of NUMBER is developed. The two parts differ in scope as well as in method.

In Part I, where both Nouns and Numerals are treated syntactical methods are used, since qualification of nouns inflected with "'S'", " ' " or "IAN" has been achieved at an earlier stage (See Table No 5) Grammatical context of the analyzed Noun or Numeral is checked up for the presence of some 'governing' or 'coordinating' element preceding the analyzed word. Prepositions, verbs, verbal nouns and numerals belong to the 'governing' group, whereas conjuctions and punctuation marks with indication 'Homogeneous' or conjunctions of comparison are considered 'coordinating'. In both cases the indication required is taken from one of the preceding words, either governing or coordinated with the analyzed one. If neither is the case, other patterns are applied, Special attention is given to Conjunctive Nouns.

As to Part II of the routine, Nouns are the only class of words analyzed here, morphological methods providing the most important information for developing the indication of number. Ifthe word is 'inflected'[1/] the number is defined as 'Plural', otherwise syntactical methods are applied.

Mention should be made of the fact, that patterns of grammatical context are solved here so as to reflect the peculiarities of Number

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1/ See Notes on Table No 5.

and Case forms in the English and Russian languages. Thus, with number indicetions, differences in classifying nouns into 'countables' and 'uncountables' in English and in Russian are taken into account. Among other idiomatic constructions in Russian that of 'Numeral + Noun' combination, where the Numeral has retained the old 'dual' government, should be pointed out. Certain peculiarities in Russian verb government are also given consideration.

Syntactically, Nouns and Numerals are classified only when used in the function of an Atribute or Subject of the sentence.

B.5. The 'Adjective Analysis' routine comes the last in the series of English Analysis routines which develop grammatical indications to be used in the Russian Synthesis routines. Information acquired of the Adjectives (and Participles) of the text includes the indications of gender, number, case, degree of comparison and short/full form. In addition to these, the indications of 'Substantivized' or 'Adverbial Adjective' are developed.

Preliminary ckeck-ups for the absence of indications required are followed by testing the morphological structure and syntactical environment of the analyzed word. The main interest of the testing procedure lies in finding out whether the analyzed word is placed in the attributive position towards some noun of the sentence[1/]. If the search is positive, it becomes very important to pick up the right noun,which in some cases is not a very easy task.

Another important search is aimed at establishing a predicative position of the analyzed word. Finally, if the search is negative,

1/ Sometimes it can be a noun of another sentence, as in case of conjunctive adjectives.

the word is qualified as 'Substantivized'.

B.6. The 'Changes of Word-Order' routine is meant to give a 'final touch' to the translated text before the Synthesis routines are applied.

English patterns of word-order which do not correspond to Russian patterns are recomposed. It is remarkable, however, that these recompositions are mostly of local character.

The most important changes of word-order, perfomed in accordance with this routine[1/], are due to the difference in the position of attributes expressed by nouns or noun combinations (See Tables No 11 and 12) as well as in the expression of negation in the English and Russian languages.

Other changes are of no particular importance.

4. CONCLUSIONS.

The heart of the whole method suggested above lies in the most careful discription of every language included in the MT system, a very detailed subsequent comparison of these descriptions being the basis of MT research.

The comparison of the English and Russian languages in the course of MT studies has proved to be more fruitful than could have been supposed, insofar as the structure of these languages has been found strikingly alike, up to a great many details. For this reason, an attempt was made to work out an Anglo-Russian MT scheme where maximum similiarities found in the structures of the two languages

[1/] Certain more specialized changes of word-order are performed at an earlier stage of Analysis (See above: Sections B.I. and B.2).

would be made use of.

Owing to this, structural transformations of the translated text have been restricted in the present scheme of MT to such minimum as omittance and insertion of just a few 'helping' words or punctuation marks and a few (local) changes of word-order.Nevertheless, the translations thus obtained are quite adequate for understanding and do not require post-editing,as can be seen in the samples cited below (See Tables Nos 1,2,3,4)

<u>**References.**</u>

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attachment to (4)

APPENDIX

NOTES ON TABLES Nos 1,2,3,4

1. SAMPLES OF TEXTS TRANSLATED IN STRICT ACCORDANCE WITH TRANSLATIONAL ROUTINES DEVISED FOR MT ARE GIVEN IN TABLES Nos 1,2,3,4. WORDS UNDERLINED WERE EITHER NOT FOUND IN THE MT DICTIONARY AT ALL, OR THEIR MEANINGS WERE DIFFERENT FROM THOSE REQUIRED IN THE PRESENT TEXTS.

NOTES ON TABLE No 5

1. THE ROUTINE OF 'AFFIX DISCARDING AND VOCABULARY SEARCH' IS GIVEN IN FULL IN TABLE No 5. THE ROUTINE DOES NOT INCLUDE:
   a) RECONSTRUCTION OF IRREGULAR VERB FORMS, AS WELL AS CERTAIN IRREGULAR FORMS OF NOUNS, NUMERALS, ADJECTIVES AND ADVERBS, STORED IN THE MT DICTIONARY;
   b) ANALYSIS OF CONTRACTED FORMS, SUCH AS "...'ll", "...'d", "...'s" IN "I'll", "we'd", "he's", ETC., SINCE THESE ARE NOT CHARACTERISTIC OF SCIENTIFIC TEXTS.
2. AS SOON AS THE ANALYZED WORD IS FOUND IN MT DICTIONARY IT IS REPLACED BY ITS VOCABULARY FORM, INDICATION 'INFLECTED' BEING DEVELOPED IN CASE THE DISCARDED AFFIX BELONGS TO THE STARRED AFFIXES.
3. INDICATION PM IS GIVEN ONLY TO THOSE PUNCTUATION MARKS, WHICH ARE MEANT TO BE ANALYZED BY THE 'PUNCTUATION MARKS ANALYSIS' ROUTINE.

NOTES ON TABLES Nos 5,6,7

1. TWO EXAMPLES OF CONTEXT ANALYSIS CARRIED OUT BY THE 'POLYSEMANTIC WORD ANALYSIS' ROUTINE ARE GIVEN IN TABLES Nos 6 AND 7.
2. THE FOLLOWING SYMBOLS ARE ACCEPTED IN THE ROUTINES: A(B,C) MEANS PASSING ON TO No. B IN THE CASE OF THE POSITIVE ANSWER, WHEREAS NEGATIVE ANSWER WILL RESULT IN PASSING ON TO No. C. OBVIOUSLY, A(B) MEANS PASSING ON TO No. B IN BOTH CASES, AND A(0) MEANS THAT THE FINAL RESULT IS ACQUIRED AND NO FURTHER SEARCH IS NECESSARY.

   BOTH FIGURES AND LETTERS, AS WELL AS THEIR COMBINATIONS, ARE USED IN THE ROUTINES IN THE MANNER EXPLAINED ABOVE GENERALLY

Repeated complex roots occur seldom in practical problems and are evaluated by trial and error, since the iteration process converges very slowly (if **at all**) in this case.

"Squaring the roots," or *Graeffe's method*,* is frequently more cumbersome than the methods outlined above, particularly in connection with complex roots, but is widely used.

Once all the roots x_i of an algebraic equation have been obtained, the results of the solution may be checked by means of *Newton's relations:*

$$\sum_{i=1}^{n} x_i = -\frac{a_{n-1}}{a_n} \qquad (1.3.3)$$

$$x_1 \cdot x_2 \cdot x_3 \cdot \ldots \cdot x_n = (-1)^n \frac{a_0}{a_n} \qquad (1.3.4)$$

For example, in Eq. (a) of this section:

$$\sum_{i=1}^{4} x_i = (1.3 + 1.49i) + (1.3 - 1.49i) + (-15 + 12.5i) + (-15 - 12.5i) = -27.4$$

$$x_1 \cdot x_2 \cdot x_3 \cdot x_4 = 1491,$$

indicating that the imaginary parts of the roots are probably slightly inaccurate.

1.4 Transcendental Equations

Any nonalgebraic equation is called a *transcendental equation*. A transcendental equation may have a finite or an infinite number of real roots, and may have no real roots at all. For example, the equation

$$\sin x = 2$$

has no real roots (Fig. 1.3) but an infinity of complex roots; the equation

$$\sin x = \tfrac{1}{2}$$

* This method is explained in detail in J. B. Scarborough, *Numerical Mathematical Analysis*, Johns Hopkins Press, Baltimore, 1930, pp. 198 ff., and in R. E. Doughtery and E. G. Keller, *Mathematics of Modern Engineering*, John Wiley & Sons Inc. New York 1936 pp. 98 ff.

TABLE 1

Кратные комплексные корни редко встречаются в практических задачах и вычисляются при помощи метода ложного положения, поскольку итерационный процесс сходится очень медленно (если вообще сходится) в этом случае. "Возведение корней в квадрат" или метод Graeffe'а, часто более громоздко, чем методы, описанные выше, особенно в связи с комплексными корнями, но широко используются. Если все корни x_i алгебраического уравнения получены, то результаты решения можно проверить при помощи соотношений Newton'а:

$$\sum_{i=1}^{n} x_i = -\frac{a_{n-1}}{a_n}$$

$$x_1 \cdot x_2 \cdot x_3 \cdot \ldots \cdot x_n = (-1)^n \frac{a_0}{a_n}$$

Например, в уравнении (а) этого раздела:

$$\sum_{i=1}^{4} x_i = (1{,}3+1{,}49i) + (1{,}3-1{,}49i) + (-15+12{,}5i) + (-15-12{,}5i) = -27{,}4$$

$$x_1 \cdot x_2 \cdot x_3 \cdot x_4 = 1491,$$

показывая, что мнимые части корней, вероятно, немного неточны.

1.4. Трансцендентные уравнения

Любое неалгебраическое уравнение называется трансцендентным уравнением. Трансцендентное уравнение может иметь конечное или бесконечное число вещественных корней и может не иметь никаких вещественных корней вообще. Например, уравнение

$$\sin x = 2$$

не имеет никаких вещественных корней (F.13), кроме бесконечного количества комплексных корней; уравнение

$$\sin x = \frac{1}{2}$$

(Salvadori „Numerical Methods in Engineering", p.13)

Table 2

He could not say much more about it, although he felt like telling them exactly what this meant. If he had been a lesser man he would have been insulted by this telegram. Being Essex, he could only assume it to be the illicit interference of someone like B. Cooke or A. Cutler. They had not wanted him on mission in the first place, and no doubt they were doing their best to make it impossible for him. He wanted to explain this to Katherine, but not a word of it could reach her. J. Aldridge, "The Diplomat" p. 227	Он не мог рассказать значительно больше об этом, хотя ему хотелось сказать им в точности, что это значило. Если бы он был более мелким человеком, он был бы оскорблен этой телеграммой. Будучи Эссексом, он мог только предположить, что это незаконное вмешательство кого-то, вроде Б. Кука или А. Катлера. Прежде всего, они не хотели, чтобы он ехал с миссией, и без сомнения они сделали все возможное, чтобы сделать это невозможным для него. Он хотел объяснить это Кэтрин, но ни одно слово из этого не могло дойти до нее.

Table 3

It was the only way, probably, that such a preposition could have been made to Soames. He was nonplussed. Conscience told him to throw the whole thing up. But the design was good, and he knew it -- there was completeness about it, and dignity; the servants' apartments were excellent too. He would gain credit by living in a house like that-with such individual features, yet perfectly well-arranged Galsworthy. "The Man of Property" Part I, Chapter VIII, p. 103	Вероятно, это был единственный способ, чтобы такое предложение можно было сделать Сомсу. Он был ошарашен. Рассудок велел ему отказаться от всей затеи. Но чертеж был хорош, и он знал это — в нем была законченность, и благородство; комнаты для слуг были также отличные. Он приобрел бы уважение, живя в доме, подобном этому — с такими индивидуальными особенностями, и все же совершенно благоустроенном.

Table 4

From childhood's hour I have not been As others were – I have not seen As others saw – I could not bring My passions from a common spring. Edgare A. Poe "Alone"	С часа детства я не был Как другие были — я не видел Как другие видели — я не мог черпать Мои страсти из общего источника

Table 5

ROUTINE FOR AFFIX DISCARDING AND VOCABULARY SEARCH

1 (29a, 2)		CHECK UP VOCABULARY FOR THE WHOLE WORD
2 (3a, 3b)		CHECK UP ANALYZED WORD FOR 'FORMULA'
3a (15b, 4a)		CHECK UP TWO FINAL LETTERS FOR 'S
b (15a, 4b)		
4a (14p, 9b)		CHECK UP FINAL LETTER FOR '
b (14a, 5a)		
5a (14b, 6a)		CHECK UP FINAL LETTER (OF REMAINDER) FOR **S***
b (14d, 30b)		
6a (16b, 7)		CHECK UP THREE FINAL LETTERS (OF REMAINDER) FOR **ING***
b (16b, 14c)		
7 (14e, 8)		CHECK UP TWO FINAL LETTERS FOR **ED*** OR **ER***
8 (15e, 9a)		CHECK UP THREE FINAL LETTERS FOR **EST***
9a (14h, 10)		CHECK UP TWO FINAL LETTERS FOR **TH***
b (15f, 35)		
10 (16e, 11)		CHECK UP THREE FINAL LETTERS FOR **IAN**
11 (14l, 12)		CHECK UP FINAL LETTER FOR **A***
12 (14m, 13)		CHECK UP FINAL LETTER FOR **I***
13 (17, 34)		CHECK UP THREE FINAL LETTERS FOR **MEN***
14a (28c)	h (28s)	DISCARD FINAL LETTER (OF REMAINDER)
b (28b)	k (28t)	
c (28d)	l (25)	
d (28a)	m (27)	
e (28n)	n (20)	
f (28p)	p (30a)	
g (28r)		
15a (28a)	d (28m)	DISCARD TWO FINAL LETTERS (OF REMAINDER)
b (30a)	e (28n)	
c (24)	f (31)	
	h (26)	
16a (21a)	d (19)	DISCARD THREE FINAL LETTERS (OF REMAINDER)
b (28k)	e (28z)	
c (21c)		
17 (28h)		CHANGE LAST BUT ONE LETTER FOR **A**
18 (28e)		ADD **IS** TO REMAINDER
19 (28h)		ADD **EX** TO REMAINDER
20 (28g)		ADD **FE** TO REMAINDER

21a (28f)		ADD **Y** TO REMAINDER
b (28h)		
c (28w)		
22a (28l)		ADD **E** TO REMAINDER
b (28v)		
23 (28h)		ADD **IE** TO REMAINDER
24 (28u)		ADD **VE** TO REMAINDER
25 (28y)		ADD **UM** TO REMAINDER
26 (28h)		ADD **ON** TO REMAINDER
27 (28h)		ADD **US** TO REMAINDER
28a (29b, 30b)	n (29a, 14f)	CHECK UP VOCABULARY FOR REMAINDER
b (29a, 6b)	p (29a, 14g)	
c (29b, 5b)	r (29a, 21b)	
d (29a, 18)	s (29c, 14k)	
e (29a, 16a)	t (29c, 22b)	
f (29a, 14n)	u (29c, 16c)	
g (29a, 16d)	v (29c, 15c)	
h (29a, 34)	w (29c, 34)	
k (29a, 22a)	y (29a, 15h)	
l (29a, 15d)	z (29b, 34)	
m (29a, 23)		
29a (35)		TAKE INFORMATION FROM VOCABULARY
b (33)		
c (34)		
30a (32)		DEVELOP INDICATION 'NOUN'
b (33)		
31 (35)		DEVELOP INDICATION 'ADJECTIVE'
32 (35)		DEVELOP INDICATION 'PLURAL'
33 (35)		DEVELOP INDICATION 'GENITIVE'
34 (35)		DEVELOP INDICATION 'UNKNOWN WORD' AND STORE ENGLISH WORD TO BE PRINTED UNALTERED IN RUSSIAN SENTENCE
35 (0, 1)		PICK UP NEXT WORD WITHOUT INDICATION 'PUNCTUATION MARK' (PM) AND CHECK IT UP FOR 'FULL-STOP'

Table 6

ADVANCE
Homonym 2 - 1. Noun, polysem.
2. Verb, polysem.

4(a,c)	CHECK UP ANALYZED WORD FOR INDICATION 'VERB'
a(b,1)	CHECK UP ANALYZED WORD FOR INDICATION 'INFLECTED', - THE AFFIX FOR ED AND FOLLOWING WORD FOR INDICATION 'NOUN'
b(II,III)	CHECK UP SELECTED NOUN FOR 'GROUP TEXT - BOOK'
c(IV,V)	CHECK UP PRECEDING WORD FOR 'GROUP IN'
I (0)	РАЗРАБОТАТЬ (Verb; + Accusative)
II (0)	ПОВЫШЕННЫЙ ТИП ('Adjective + + Noun' combination; Masculine, Singular, Attribute in postposition)
III (0)	СОВЕРШЕННЫЙ (Adjective)
IV (0)	ЗАРАНЕЕ (Adverb of place)
V (0)	УСПЕХ (Noun, masculine)

Table 7

DESIGN
Homonym 2 - Verb, Polysem.

69(I,a)	CHECK UP NEAREST FOLLOWING NOUN (*2a), OR NEAREST PRECEDING NOUN (*2b) FOR 'GROUP CONSTRUCTION' OR 'GROUP METHOD'
a(II,III)	CHECK UP NEAREST FOLLOWING PREPOSITION FOR INDICATION PPV, OR PD, OR PA*)
I(0)	РАЗРАБОТАТЬ (Verb; + Accusative)
II(0)	ПРЕДНАЗНАЧИТЬ (Verb; + Accusative)
III(0)	ОБОЗНАЧИТЬ (Verb; + Accusative)

*) PPV: Preposition of Passive Voice
PD: Preposition of Direction
PA: Preposition of Aim

Table 8

MT ROUTINES IN THE ORDER OF THEIR APPLICATION

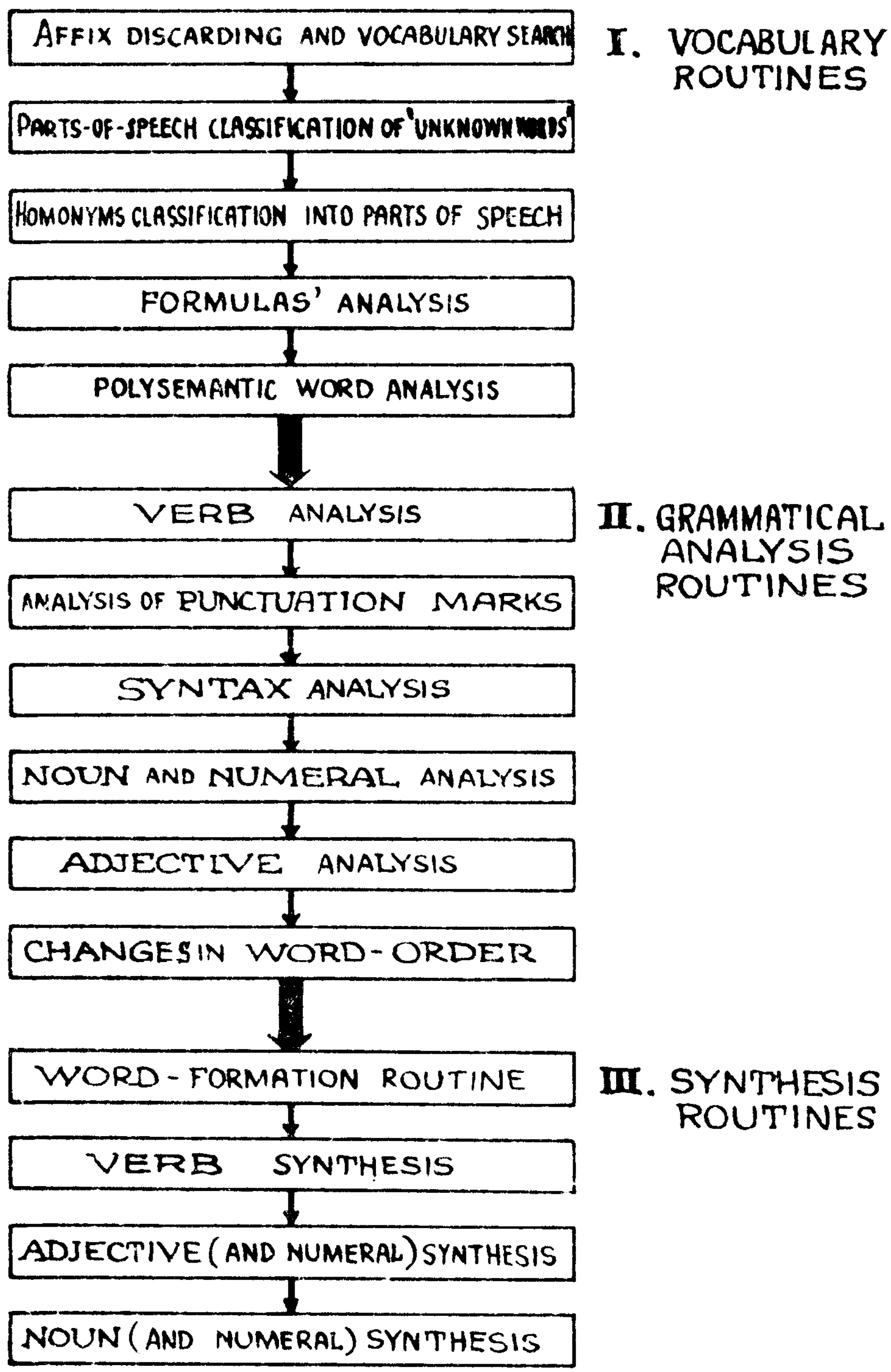

Table 9

PATTERNS OF GRAMMATICAL CONTEXT USED IN VERB ANALYSIS

Preceding word (selected) is					
1. VERB	+ analyzed verb	1A	selected verb is	Auxiliary I	
		1B		Auxiliary II	
		1C		Half-auxiliary	
		1D		Notional	
		1E		Modal	
		1F		Half-modal	
		1G		Auxiliary III	
2. NOUN	+ analyzed verb	2A Verb +	selected noun		
		2B Verbal particle +			
		2C CI +			
3. ADJECTIVAL	+ analyzed verb	3A	selected adjectival is	Pronominal adjective	
		3B		Preposition	
		3C		Numeral	
4. CONJUNCTION	+ analyzed verb	4A	selected conjunction is	CI	
		4B		CH	
		4C		PM indicated	'bracket'
		4D		PM indicated	'comma'
		4E		PM indicated	'semicolon'
		4F		PM indicated	'full-stop'
5. ADVERBIAL	+ analyzed verb	5A	selected adverbial is	Adverb	
		5B		Indefinite particle	5B-1 Verb + selected adverbial
					5B-2 Noun + selected adverbial
					5B-3 Adjective + selected adverbial

Test 951 — Table 10

E	They	insisted	on	a	parallelism	between
N°E	1E	2E	3E	4E	5E	6E
e	Noun Pronominal Personal Plural Nominative	Verb	Homonym 4 Polysem.	Adjective Pronominal Indefinite article	Noun Abstract	Preposition
R1	ОН	НАСТАИВАТЬ	НА	NT	ПАРАЛЛЕЛИЗМ	МЕЖДУ
r	Noun Pronominal — Subject Plural Nominative	Verb + Accusative — Predicate Past Plural 3d Person	Preposition + Prepositional case	Adjective D_R	Noun Masculine — Singular Prepositional Case	Preposition + Ablative
N°R	1R	2R	3R		4R	5R
wo						
R2	ОН	НАСТАИВАТЬ	НА		ПАРАЛЛЕЛИЗМ	МЕЖДУ
R	**ОНИ**	**НАСТАИВАЛИ**	**НА**		**ПАРАЛЛЕЛИЗМЕ**	**МЕЖДУ**

Test 951 (continued)

E	'arithmetical'	and	'general'	algebra	so	rigid	
N°E	7E	8E	9E	10E	11E	12E	
e	Adjective	Conjunction Polysem.	Adjective	Noun Mathematical term	Adverb D	Adjective	
R1	„АРИФМЕТИЧЕСКИЙ"	И	„ОБЩИЙ"	АЛГЕБРА	ТАК	ЖЕСТКИЙ	,
r	Adjective — Feminine Singular Ablative	CH	Adjective — Feminine Singular Ablative	Noun Mathematical term Feminine — Singular Ablative	Adverb D	Adjective — Adverbial D	CI SD
N°R	6R	7R	8R	9R	10R	11R	12R
wo							
R2	„АРИФМЕТИЧЕСКИЙ"	И	„ОБЩИЙ"	АЛГЕБРА	ТАК	ЖЕСТКИЙ	,
R	**„АРИФМЕТИЧЕСКОЙ"**	**И**	**„ОБЩЕЙ"**	**АЛГЕБРОЙ**	**ТАК**	**ЖЕСТКО**	**,**

Test 951 (continued) — Table 10 (continued)

E	that	,	if	it	could	be	maintained	,	it	would
№E	13E	14E	15E	16E	17E	18E	19E	20E	21E	22E
e	Adjective* Pronominal Polysem.		Conjunction	Noun Pronominal Impersonal Singular Polysem	Verb Modal Past Predicate	Verb Auxiliary I Polysem.	Verb	Com	Noun Pronominal Impersonal Singular Polysem.	Verb Auxiliary of mood Modal Predicate
R1	ЧТО	,	ЕСЛИ	ЭТОТ	МОЧЬ	БЫТЬ	СОХРАНИТЬ	,	ЭТОТ	NT
r	CI	CI SB	CI	Adjective Pronominal / Neuter Singular Accusative [Subject*]	Verb / Predicate Present Impersonal D	Verb / Subjunctive Impersonal Predicate D	Verb + Accusative / Perfective Subject to be converted into direct object Infinitive Predicate	CI SE	Adjective Pronominal Neuter Singular Nominative Subject	Verb Predicate D
№R	13R	14R	15R	16R	17R	18R	19R	20R	21R	
wo										
R2	ЧТО	,	ЕСЛИ	ЭТОТ	МОЧЬ	БЫТЬ	СОХРАНИТЬ	,	ЭТОТ	
R	**ЧТО**	**,**	**ЕСЛИ**	**ЭТО**	**МОЖНО**	**БЫЛО БЫ**	**СОХРАНИТЬ**	**,**	**ЭТО**	

Test 951 (continued)

E	effectively	destroy	the	generality	;	and	they
№E	23E	24E	25E	26E	27E	28E	29E
e	Adverb* D	Verb	Adjective Pronominal Definite article Polysem.	Noun Abstract	Semicolon	Conjunction Polysem	Noun Pronominal Personal Plural Nominative
R1	СУЩЕСТВЕННЫЙ	РАЗРУШИТЬ	NT	ОБЩНОСТЬ	;	И	ОН
r	Adverbial Adjective D	Verb + Accusative / Predicate Subjunctive Neuter Singular	Adjective D_R	Noun Feminine / Singular Accusative	Semicolon Stop	CI	Noun Pronominal / Subject Plural Nominative
№R	22R	23R		24R	25R	26R	27R
wo							shift
R2	СУЩЕСТВЕННЫЙ	РАЗРУШИТЬ		ОБЩНОСТЬ	;	И	КАЗАТЬСЯ
R	**СУЩЕСТВЕННО**	**РАЗРУШИЛО БЫ**		**ОБЩНОСТЬ**	**;**	**И**	**КАЖЕТСЯ**

Test 951 (continued)

Table 10 (continued)

E	never		seem		to	have	realised	fully		that
NºE	30E		31E		32E	33E	34E	35E		36E
e	Adverb D		Link- -verb		Homonym 4 Polysem.	Verb Auxiliary II Polysem.	Verb	Adverb D		Adjective* Pronominal Polysem.
R1	НИКОГДА	НЕ	КАЗАТЬСЯ	,	ЧТО	NT	ОСОЗНАВАТЬ	ПОЛНОСТЬЮ	,	ЧТО
r	Adverb D	Particle D	Verb + Ablative Predicate Present Singular 3d person	CI SD	CS	Verb Predicate Present D	Verb + Accusative Predicate Past Plural 3d person	Adverb D	CI SD	CI
NºR	28R	29R	30R	31R	32R		33R	34R	35R	36R
wo	shift	shift	PUT COMBINATION (30R)+(31R)+(32R) before (27R)			shift				
R2	,	ЧТО	ОН		НИКОГДА	НЕ	ОСОЗНАВАТЬ	ПОЛНОСТЬЮ	,	ЧТО
R	,	**ЧТО**	**ОНИ**		**НИКОГДА**	**НЕ**	**ОСОЗНАВАЛИ**	**ПОЛНОСТЬЮ**	,	**ЧТО**

Test 951 (continued)

E	a				formula		true	with	one
NºE	37E				38E		39E	40E	41E
e	Adjective Pronominal Indefinite article Polysem.				Noun Mathematical term		Homonym 3 Polysem.	Homonym 4 Polysem.	Numeral* Polysem.
R1	NT				ФОРМУЛА		ВЕРНЫЙ	ПРИ	ОДИН
r	Adjective D_R		CI SD		Noun Mathematical term Feminine Subject Singular Nominative	CA^b	Adjective Feminine Singular Nominative	Preposition + Prepositional Case	Adjective Feminine Singular Prepositional Case
NºR			37R		38R	39R	40R	41R	42R
wo	SHIFT	SHIFT OF COMBINATION (46E – 49E) PRECEEDS INSERTION OF COMMA (37R)							
R2	ВПОЛНЕ	ВОЗМОЖНЫЙ	,	ЧТО	ФОРМУЛА	,	ВЕРНЫЙ	ПРИ	ОДИН
R	**ВПОЛНЕ**	**ВОЗМОЖНО**	,	**ЧТО**	**ФОРМУЛА**	,	**ВЕРНАЯ**	**ПРИ**	**ОДНОЙ**

Test 530 — **Table 11**

E	Use	the	Lagrange	variation	–	of	–	parameters
NºE	1E	2E	3E	4E	5E	6E	7E	8E
e	Verb Polysem.	Adjective Pronominal Definite article Polysem.	Noun Animated Proper Masculine Singular	Noun Mathematical term Polysem.	Hyph	Preposition Polysem.	Hyph	Noun Mathematical term
R1	ПРИМЕНИТЬ	NT	ЛАГРАНЖ	ВАРИАЦИЯ		NT		ПАРАМЕТР
r	Verb Accusative Predicate Imperative	Adjective D_R	Noun Animated Proper Masculine Singular Genitive	Noun Mathematical term Feminine Singular Genitive	Hyph CD_R	Preposition + Genitive	Hyph CD_R	Noun Mathematical term Masculine Plural Genitive
NºR	1R		2R	3R				4R
wo			PUT (2R) AFTER (4R)	PUT COMBINATION (3R) + (4R) + (2R) AFTER (5R)				
R2	ПРИМЕНИТЬ		МЕТОД	ВАРИАЦИЯ				ПАРАМЕТР
R	**ПРИМЕНИТЕ**		**МЕТОД**	**ВАРИАЦИИ**				**ПАРАМЕТРОВ**

Test 530 (continued)

E	method	to	solve	the	inhomogeneous	n^{th}	–	order
NºE	9E	10E	11E	12E	13E	14E	15E	16E
e	Noun	Homonym 4 Polysem.	Verb	Adjective Pronominal Definite article Polysem.	Adjective	Adjective Ordinal Formula	Hyph	Noun Polysem.
R1	МЕТОД	К	РЕШИТЬ	NT	НЕОДНОРОДНЫЙ	n ый		ПОРЯДОК
r	Noun Masculine Singular Accusative	Preposition +Dative	Verb +Accusative Verbal Noun Singular Dative	Adjective D_R	Adjective Masculine Singular Genitive	Adjective Masculine Singular Genitive	Hyph CD_R	Noun Masculine Singular Genitive
NºR	5R	6R	7R		8R	9R		10R
wo	SHIFT					PUT COMBINATION (9R) + (10R) AFTER (12R)		
R2	ЛАГРАНЖ	К	РЕШИТЬ		НЕОДНОРОДНЫЙ	ДИФФЕРЕНЦИАЛЬНЫЙ		
R	**ЛАГРАНЖА**	**К**	**РЕШЕНИЮ**		**НЕОДНОРОДНОГО**	**ДИФФЕРЕНЦИАЛЬНОГО**		

Test 530 (continued)

Table 11 (continued)

E	differential	equation		$y^{(n)}+a_1y^{(n-1)}+\dots+a_ny=f(t)$ (a)	.
N°E	17E	18E		19E	20E
e	Homonym 1	Noun		Formula	Full-stop
R1	ДИФФЕРЕНЦИАЛЬНЫЙ	УРАВНЕНИЕ		$y^{(n)}+a_1y^{(n-1)}+\dots+a_ny=f(t)$ (a)	.
z	Adjective Mathematical term Neuter Singular Genitive	Noun Neuter Singular Genitive		Formula D_E	Full-stop
N°R	11R	12R		13R	14R
WO	SHIFT	SHIFT			
R2	УРАВНЕНИЕ	$n^{ый}$	ПОРЯДОК	$y^{(n)}+a_1y^{(n-1)}+\dots+a_ny=f(t)$ (a)	.
R	**УРАВНЕНИЯ**	**$n^{ого}$**	**ПОРЯДКА**	**$y^{(n)}+a_1y^{(n-1)}+\dots+a_ny=f(t)$ (a)**	**.**

Test 885

Table 12.

E	The	examples	of	Chapters	V	and	VI	,	in	which	Lagrange's
N°E	1E	2E	3E	4E	5E	6E	7E	8E	9E	10E	11E
e	Adjective Pronominal Definite article	Noun	Preposition	Noun	Formula	Conjunction	Formula	Com	Homonym 4	Adjective Conjunctive	Noun Animated Proper Masculine Singular
	Polysem.		Polysem.			Polysem			Polysem		
R1	NT	ПРИМЕР	ИЗ	ГЛАВА	V	И	VI	,	В	КОТОРЫЙ	ЛАГРАНЖ
z	Adjective D_R	Noun Masculine [Subject *] Plural Accusative	Preposition + Genitive	Noun Feminine Plural Genitive	Formula D_E	CH D_E	Formula D_E	CI SB	Preposition + Prepositional case	Adjective [illegible]	Noun Animated Proper Masculine Singular Genitive
N°R	1R		2R	3R	4R	5R	6R	7R	8R	9R	10R
WO											PUT (10R) AFTER (11R)
R2		ПРИМЕР	ИЗ	ГЛАВА	V	И	VI	,	В	КОТОРЫЙ	УРАВНЕНИЕ
R		**ПРИМЕРЫ**	**ИЗ**	**ГЛАВ**	**V**	**И**	**VI**	**,**	**В**	**КОТОРЫХ**	**УРАВНЕНИЯ**

Test 885 (continued) — Table 12 (continued)

E	equations	are	used	,	can	be	treated	without
N°E	12E	13E	14E	15E	16E	17E	18E	19E
e	Noun	Verb Auxiliary I Predicate Present Plural Polysem.	Homonym 2 Polysem.	Com	Verb Modal Predicate Present Polysem.	Verb Auxiliary I Polysem	Verb	Preposition Polysem.
R1	УРАВНЕНИЕ	NT	ИСПОЛЬЗОВАТЬ		МОЧЬ	NT	РАССМАТРИВАТЬ	БЕЗ
τ	Noun Neuter --- Plural Nominative Subject	Verb +Nominative --- Predicate Present Plural D	Verb + Accusative --- Predicate Present Passive Plural 3d person	CI SE	Verb --- Predicate Present Impersonal D	Verb --- Predicate Infinitive D	Verb + Accusative --- Predicate Infinitive Subject to be converted into direct object	Preposition + Genitive
N°R	11R		12R	13R	14R		15R	16R
wo	SHIFT							
R2	ЛАГРАНЖ		ИСПОЛЬЗОВАТЬ	,	МОЧЬ		РАССМАТРИВАТЬ	БЕЗ
R	**ЛАГРАНЖА**		**ИСПОЛЬЗУЮТСЯ**	,	**МОЖНО**		**РАССМАТРИВАТЬ**	**БЕЗ**

Test 885 (continued)

E	reference	to	the	Lagrangian	method	.	
N°E	20E	21E	22E	23E	24E	25E	
e	Noun	Homonym 4 Polysem.	Adjective Pronominal Definite article Polysem.	Noun Animated Proper Masculine Singular	Noun	Full-stop	
R1	ССЫЛКА	НА	NT	ЛАГРАНЖ	МЕТОД	.	
τ	Noun Feminine --- Singular Genitive	Preposition + Accusative	Adjective D_R	Noun Animated Proper Masculine Singular --- Genitive	Noun Masculine --- Singular Accusative	Full-stop	
N°R	17R	18R		19R	20R	21R	
wo				PUT (19R) AFTER (20R)	SHIFT		
R2	ССЫЛКА	НА		МЕТОД	ЛАГРАНЖ	.	
R	**ССЫЛКИ**	**НА**		**МЕТОД**	**ЛАГРАНЖА**	.	

5

ANALYSIS OF THE WORKING PRINCIPLES OF SOME SELF ADJUSTING SYSTEMS IN ENGINEERING AND BIOLOGY

by

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The concept of control process arises in various branches of natural sciences and possesses a number of common features which are studied by cybernetics.

The most highly organized controlling processes are those related to higher nervous activity of animals and man. The study of these processes from the point of view of general cybernetics by revealing the laws of the corresponding control algorithms is of great interest. It gives an understanding of the nature of higher nervous activity, enriches the theory of control algorithms and helps to create new classes of effective algorithms. A considerable number of publications have been published by now, concerned with attempts to model various features of the control processes in the central nervous system. These include investigations on modelling conditional reflexes (1), (10), on the creation of various models of mice, turtles, etc. which imitate the behaviour of the live creatures (3), (6), (7), (8), works on the mathematical theory of learning (4), (9), etc.

It is more difficult to indicate publications which develop consistently a cybernetical approach to the study of actual questions of the physiology of higher nervous activity and to revealing the features of the corresponding control algorithms.

It is appropriate at this point to quote Academician I.P. Pavlov (2) who pointed out the necessity of employing mathematical analysis in studying processes of higher nervous activity.

This paper discusses questions of algorithm creation for complex control processes reflecting the features of formation of conditional reflex chains.

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Para.1. GENERAL CONTROL SYSTEM

Any controlling process infers the existence of two devices - one controlling device and the other controlled - which exchange information. The complexity of the controlling device depends on the quantity of information it has to treat over a definite interval of time. The controlling device can employ the information about the controlled device in various ways. In the general case the controlling device may receive information as to the influence of the environment on the controlled device, characterized at each moment by a signal $\mathcal{F}(t)$, and information as to the result of the action of the controlled device, characterized by a signal $y(t)$. On the basis of this information the controlling device produces a signal $x(t)$ which acts on the controlled device. The algorithm of producing the signal $x(t)$ is determined by the condition that the signal $y(t)$ differs sufficiently little in some sense from a predetermined function $\tilde{y}(t)$. Thus the result of the action of the controlled device is a function of the controlling influence $x(\tau)$ and of the influences of the environment $\mathcal{F}(\tau)$ and $\psi(t)$, where $\tau \leq t$:

$$y(t) = \mathcal{Y}(x(\tau), \mathcal{F}(\tau), \psi(t)) \qquad (1)$$

here $\psi(t)$ describes the part of the influence of the environment concerning which no information is received.

In its turn, the controlling influence is produced by the controlling device by realization of the algorithm A which determine $x(t)$ by the values of $\mathcal{F}(\tau)$ and $y(\tau)$ at $\tau < t$. The signals $\mathcal{F}(t)$ and $\psi(t)$, generally, are accidental. If $\mathcal{F}(t)$ and $\psi(t)$ are known functions the possibility of obtaining the required signal $\tilde{y}(t)$ at the output of the controlled device depends only on the permissible arbitrariness in selecting $x(t)$. In the contrary case, when the values of $\psi(t)$ are independent for various t and the right side of (1) depends substantially on the value of $\psi(t)$ the result $y(t)$ will inevitably possess a certain minimum degree of indefiniteness which cannot be lowered by virtue of the controlling influences. The most interesting case is the intermediate one when the values of $\mathcal{F}(t)$ and $\psi(t)$ are correlated at various moments of time.

In the following we assume that the time t runs through a discrete sequence of values (for instance all the integral values) and the indefiniteness of the signal is characterized by its entropy (see /5/).

In the case of the process of formation of a conditional reflex the above-described scheme of control can be integrated as follows.

The signal $\mathcal{F}(t)$ represents information on conditional stimuli; the signal $y(t)$ brings information on the reinforcements (unconditional stimuli), $x(t)$ is a characteristic of the occurrence of non-occurrence of the reaction (reflex), $\psi(t)$ characterizes the indefiniteness in the correspondence between the reinforcement and the stimuli.

The complexity of the controlling process is characterized by the total entropy H of information of the signals at various moments of time.

An important characteristic of the process is the value

$$H_{\infty} = \lim_{T \to \infty} \frac{1}{T} H\,(y(t),\ \mathcal{F}(\tau))_{t-T<\tau<t} \qquad (2)$$

It can be shown that the value H_{∞} determines the minimum degree of indefiniteness (entropy) of the value $y(t)$ per unit time which can be obtained by an optimum method of control, i.e. by selection of the algorithm A.

Another important class of controlling systems are systems for which the following limit exists

$$\tilde{H} = \lim_{T \to \infty} [H(y(t),\ \mathcal{F}(\tau))_{t-T<\tau<t} - TH_{\infty}] \qquad (3)$$

The value $T = \tilde{T}$, at which the expression in brackets in (3) differs sufficiently little from its limit $\tilde{H}$, characterizes the time interval during which the information must be received to produce the controlling signal $x(t)$ ensuring a sufficiently good quality of control, i.e. entropy $H(y(t))$ close to $\tilde{H}$. The value $\tilde{H}$ itself characterizes the time necessary to produce the control algorithm A, i.e. the "learning time" $\tilde{t}$ according to the law

$$\tilde{t} \sim \tilde{T}\, 2^{\tilde{H}} \qquad (4)$$

It is assumed that $\Psi(t)$ is a stationary stochastical process satisfying certain special limitations.

To study the dynamics of formation of chains of conditional reflexes it is important to consider the case where $\mathcal{F}(t)$ is a chain of successively acting conditional stimuli ($b_1, b_2, \ldots b_K$) $y(t)$ is a chain of successively acting reinforcing factors ($c_1, c_2, \ldots, c_K$), determined by the sequence of actions $x(t)$ ($a_1, a_2, \ldots a_K$).

We may consider a scheme in which each successive stimulus b_i is at the same time a reinforcing factor $b_i = c_{i-1}$ and the factor c_K is the final reinforcing factor (food, removal of pain).

This scheme decreases substantially. The next step in reducing $\tilde{H}$ is to establish the correlated subsequences of A actions in combination with stimuli and to work out an algorithm from such pre-set subsequences of actions.

Experimental study of systems of conditional reflexes in animals enables disclosure of some algorithms of the work of the brain. Analysis of these algorithms is possible on the basis of the theoretical conceptions considered above. The schemes of these experiments and their qualitative interpretation is given in the following paragraphs.

Para.2. EXPERIMENTAL INVESTIGATION OF CONTROL ALGORITHMS IN FORMING CHAINS OF CONDITIONAL REFLEXES

The method used by us can be characterized as a version of the conditional reflex procedure developed by I.P. Pavlov (2). A valuable contribution to the study of this problem were the works of L.G. Voronin (12).

In the course of the experiment definite systems of outer regularities were created artificially in the experimental environment surrounding the animal. For instance, the experimenter would set the scheme:

bell → $\boxed{\text{jump to pedestal No.1}}$ → whistle $\boxed{\text{press pedal}}$ → food

According to this scheme the experimenter was to introduce various stimuli depending on the movements of the animal. In this way the experimenter played the part of the environment.

In the course of the experiment, by using various versions of the procedure studies could be made of the laws relating to the formation of a new working programme (chain of conditional reflexes) in the animal, enabling it to obtain food under these experimental conditions. The animal would ascertain the regularities of the environment created artificially by the experimenter (recorded preliminarily on paper) and unknown to it. On this basis it would develop its optimum working programme under the given conditions. This procedure, which was used by us in experiments with a human being as well, reveals the complex picture of interaction between the environment and the organism, observed in the process of developing new working programmes. It thus becomes possible to disclose a certain sequence of operations by means of which the new working programme can be developed. This sequence includes both a definite system in accomplishing moving reactions and definite principles in estimating and utilizing the information received from the environment.

On the whole, we can speak of a definite learning algorithm, i.e. of a definite sequence of operations by means of which a new working programme can be developed. It should be emphasized that in this case we do not mean an algorithm of simple behaviour of animals, but an algorithm of a special higher category, an algorithm of a higher order, through which animals can develop various new forms of algorithmic behaviour in any new situations of the environment.

The experimental procedure enables various outer situations to be created. These situations may differ both in the complexity of the regularities artificially created and in the probability of accidental (not depending on the actions of the animal) appearance of various stimuli. Along with rigid regularities accidental coincidences of stimuli may also be included in the outer situation.

Under these various conditions of the artificially created environment various algorithms could be revealed on which the formation of new working programmes are based.

The procedure employed also enabled the study of algorithms in cases where the animals had already received partial or complete information of the experimental situation.

Investigations, carried out according to the above described procedure established that the chain of reflexes could be developed on the basis of double reinforcement by the successive addition of new links. In working out each new link of the chain one of the conditional stimuli of the earlier developed chain links is employed as an immediate reinforcement. The entire system after completion was reinforced by food.

The algorithm described ensures the appearance of new working programmes (behaviour algorithms) of the animal under various new conditions. It should be emphasized that this algorithm involves estimation and selection from all the information received by the brain, of that part of the information which is needed to build an optimum working programme. One of the criteria of estimating information is the principle of recurrent coincidence of two signals discovered by I.P. Pavlov. This criterion is quite reliable, because recurring coincidence may serve as a proof of the fact that in our case the organism has to deal with real laws of the outer world and not with accidental coincidence of two stimuli. The criterion of usefulness of information is at first a temporary coincidence of this information with food and afterwards with a conditional signal, in its turn, previously related to food.

It is important to stress that in the course of development of a system of reflexes new "landmarks" keep arising which serve to estimate both the usefulness of various movements and the usefulness of the information received by the control system. These "landmarks" include all conditional stimuli of newly developed conditional reflexes. The appearance of new intermediate "landmarks" may be of great importance for developing new chains of conditional reflexes (new working programmes). The importance of the appearance of these "landmarks" consists in artificial limitation of the entropy by decreasing the number of situations considered, i.e. by the eliminating the necessity of fully considering all the situations.

Para.3. MORE COMPLEX CONTROL ALGORITHMS

The case considered above was an extremely simple one. As is known, much more complex programmes have to be used in cybernetic systems. Very often a possibility is provided to switch the work of a system from one programme to another depending on the results of the work of the system.

There arises the question whether such complex programmes can be developed in self-organizing systems. This involves both the problem of finding the corresponding algorithms and the question of their realization in cybernetic machines. The experiments were made according to the described procedure, but the nature of the regularities of the environment set up in the course of the experiment were more complex. For instance, a stimulus was introduced making accomplishment of the reflex chain impossible. Or such a system of environment regularities would be set up by virtue of which two stimuli would have to be present simultaneously for any definite action to be accomplished.

The corresponding diagrams are:

$$\begin{array}{l} \quad a_4\ \text{б}_8\ a_8\ \text{б}_{10}\ a_2 \longleftarrow C\ (\text{food}) \\ a_3\,\text{б}_1\,a_4\,\text{б}_7 \longleftarrow \text{б}_6 \end{array} \qquad (5)$$

In this scheme the presence of б_6 made accomplishment of the main reflex system impossible. At the same time there was a potential possibility of finding a way of eliminating б_6 by means of the reflex chain $a_3\,\text{б}_1\,a_4\,\text{б}_7$ or

$$\left.\begin{array}{l} a_4\ \text{б}_8\ a_6\ \text{б}_{10} \\ a_3\ \text{б}_1\ a_4\ \text{б}_7 \end{array}\right\} a_2 \longleftarrow C(\text{food}) \qquad (6)$$

In this case to accomplish the action a_2 two signals б_{10} and б_7 had to be present. Such systems of interrelations are often encountered. It was established that when animals were placed under such experimental conditions they worked out the corresponding system of reflexes. The laws of development of these systems of reflexes were studied. Particularly, it was established that exclusion of the stimulus б_6 (diagram (5)) and introduction of б_7 and б_{10} (diagram (6)) may serve as a basis for the formation of a new chain of reflexes. Development of this chain of reflexes no longer required direct reinforcement by food. These chains thus differed from ordinary food reflex chains. They possessed a certain "autonomy". (As is known, the elements of a food chain disappear quickly if they are not reinforced with food).

On the basis of these experiments certain conclusions can be drawn as to the nature of the algorithms lying at the basis of the formation of complex systems of conditional reflexes. An important element of these algorithms is the appearance of a new trial of all possible movements in conditions of the presence of stimulus σ_6 (diagram (5)) and the absence of stimuli σ_{10} and σ_7 (diagram (6)).

If, as a result of any movement, stimulus σ_6 disappeared or stimuli σ_{10} or σ_7 appeared, a new "autonomous" chain of reflexes began to form, this chain being no longer related directly to food. In these experiments it was demonstrated that systems of conditional reflexes may have a very complex structure.

Para. 4. ALGORITHMS UTILIZING PREVIOUSLY DEVELOPED CHAINS OF CONDITIONAL REFLEXES

The most general case is the situation when the system possesses partial information about the controlled object. Evidently, in this case the algorithm of the work of the system should make it possible to draw precisely the information needed at that particular moment from the memory and to include it in a strictly definite part of the newly formed working programme.

This system must possess a memory and mechanisms for selection of the necessary information.

Let a certain control system have a definite number of previously developed working programmes. There is a certain concrete situation (a set of signals $\sigma_1, \sigma_2, \sigma_3$ etc. entering the system). Under these conditions the system is confronted by the task of developing a new programme of work by means of which it can accomplish a certain new result. It may also be required that certain dangerous states of the controlled object should be evaded.

The work of the controlled system must evidently result in a certain new programme which should correspond to the given concrete situation ($\sigma_1, \sigma_2, \sigma_3$) and accomplish the same purpose. In other words a certain new expedient form of behaviour must be worked out.

This new programme arises on the basis of a retreatment of previously accumulated information. In the course of retreatment of all the information stored in the memory that part of it must be selected which may be of use in achieving the end and can be utilized in the given concrete situation represented by a definite set of signals ($\sigma_1, \sigma_2, \sigma_3, \ldots$).

The information is stored in the form of a large number of work programmes of various kinds and developed at different occasions.

To analyze this scheme various kinds of complex conditional reflex systems reinforced by food, pain disappearance stimuli, etc. were developed in the animals (13).

In the course of the experiment some new demand was brought up. For instance, thirst was caused in dogs and a certain new set of external stimuli was started.

Under these conditions new forms of behaviour developed in the animal connected with the reception of water, including definite sections of previously developed food and defensive reflex chains. Experiments of this kind made it possible to study the algorithms of formation of new working programmes when partial or complete information on the controlled object is available. During this experiment the previously developed reflex systems and the experimentally created situation (set of signals) were known exactly, so that each part of the new working programme could be traced to the corresponding previously developed chain. This made it possible to observe the process of formation of the new programme.

By varying the form of the experiment certain essential laws could be established characterizing the formation of new reflex systems on the basis of newly accumulated information.

It was found that when any definite situations were created new systems of conditional reflex reactions arose immediately in the animals without additional development.

These working programmes corresponded each time exactly to the experimental environment created artificially and resulted in satisfaction of the new demand of the animal (reception of water).

These acts of behaviour could not be reduced to the sum total of stimuli included in the set. Each act of behaviour was a unified integral system and represented an integral reaction of the organism accomplished in response to the entire set of signals presented. The presence or absence of any one stimulus in the set entirely alters the newly arising system of conditional reflexes.

It was found that under ordinary conditions animals would not react at all to conditional stimuli, though the reflex system was stably developed. In creating thirst (excitement of the drinking centre) conditional reflex reactions were observed to appear in response to definite conditional stimuli. Here, activation of the individual reactions was of a selective nature. The order of distribution of this activity depended on the whole set of signals employed. The animals' reactions were always of an integral nature.

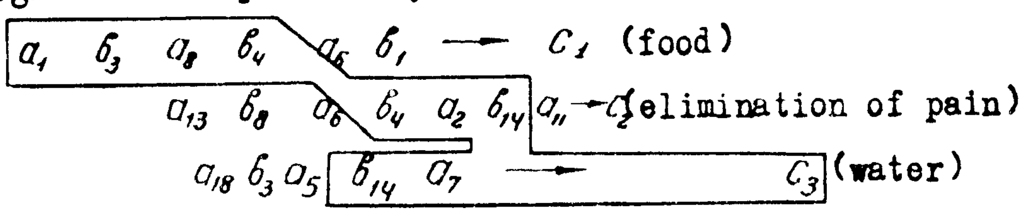
In studying this phenomenon definite laws were established. A set of special "starting" conditional irritators were detected which, themselves not causing any moving reaction, put into action a certain section of the system. A certain order of subordination was detected in the action of these signals. One of the stimuli A (a stimulus of the highest category) would put a definite sector of a previously developed reflex system into an active state. Other stimuli ($B_1, B_2, B_3, \ldots$) would start up individual parts of this sector, but their action could manifest itself only after stimulus A had been brought into play. A third group of stimuli ($C_1, C_2, C_3, \ldots$) would start up individual reflex chains. Their action was found to be possible only after A and B_1 had been put into action.

Then the following regularities were discovered. If thirst was caused, the first conditional reflexes to become active were those of the drinking chains. And the first of these conditional reflexes to go into action were those immediately connected with unconditional reinforcement (water). If any of these conditional stimuli were present in the experimentally created environment the other conditional reflexes would not become active. But if these stimuli were absent other conditional reflex reactions began to become active. If at least one common stimulus of two unlike reflex chains (the food and the drink systems) was present the animal would begin to respond to the conditional irritators of the food reflex system. If a common irritator of the food and the defence chains was present a certain section of the defence reflex chain also became active. If there were no signals common to the two unlike reflex chains no conditional reflex reactions of those systems were observed to become active.

The above-described process of the consecutive rise of conditional reflex responses to a definite group of stimuli depended also, as was indicated above, on the presence in the environment of definite "starting" stimuli. This caused the phenomenon of subordination just described. Any definite chain of reflexes could become active only if the corresponding starting signals were present.

On the basis of modern data of neurology and cybernetics the following hypothesis can be put forth to explain the facts presented above. When thirst is caused the resulting excitation process begins to spread over systems of previously developed conditional reflex ties. This lowers the excitement threshold of the corresponding first elements. If, as the excitation spreads, another excitation caused by an external conditional stimulus arrives at the same nerve cells the two excitations add up and result in the corresponding conditional reflex movement. The necessary information is selected as a result of these processes.

The union of unlike reflex chains (food and drink, and defence, etc.) can be explained in a similar manner. If a common stimulus is present the excitation process may spread from one reflex system to another. Synthesis of the new working programme is explained by the following diagram:



It can be seen from this diagram that the algorithm of union of the systems by the principle of a common stimulus may result in the synthesis of a new system of reflexes $a_1\,\text{б}_3\,a_8\,\text{б}_4\,a_2\,\text{б}_{14}\,a_7 \longrightarrow C_3$ (water) serving for the procurement of water and consisting of various sections of previously developed systems.

The brain work algorithms connected with the utilization of previously accumulated information which we have studied provide for rapid formation of new forms of behaviour.

Analysis of the principles of formation of conditional reflex chains contributes to progress in the study of questions of the physiology of a higher nervous activity in that it helps to establish the structure of the algorithms controlling the complex behaviour of higher animals and offers an opportunity of finding ways of economically realizing controlling algorithms in cybernetics systems.

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